PISA 2022 Subject Seminar Mathematics Literacy

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Content

- What is mathematical literacy?
- What are the PISA results
- What did the results tell?
- How could we improve our mathematics education?

Definition of Mathematical Literacy

Mathematical literacy is an individual's capacity

- to reason mathematically and
- to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts.
- It includes concepts, procedures, facts and tools to describe, explain and predict phenomena.
- It assists individuals to know the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective 21^{st} Century citizens.

The Framework in PISA

Figure 2. PISA 2022: the relationship between mathematical reasoning, the problem solving (modelling) cycle, mathematical contents, context and selected 21st century skills.

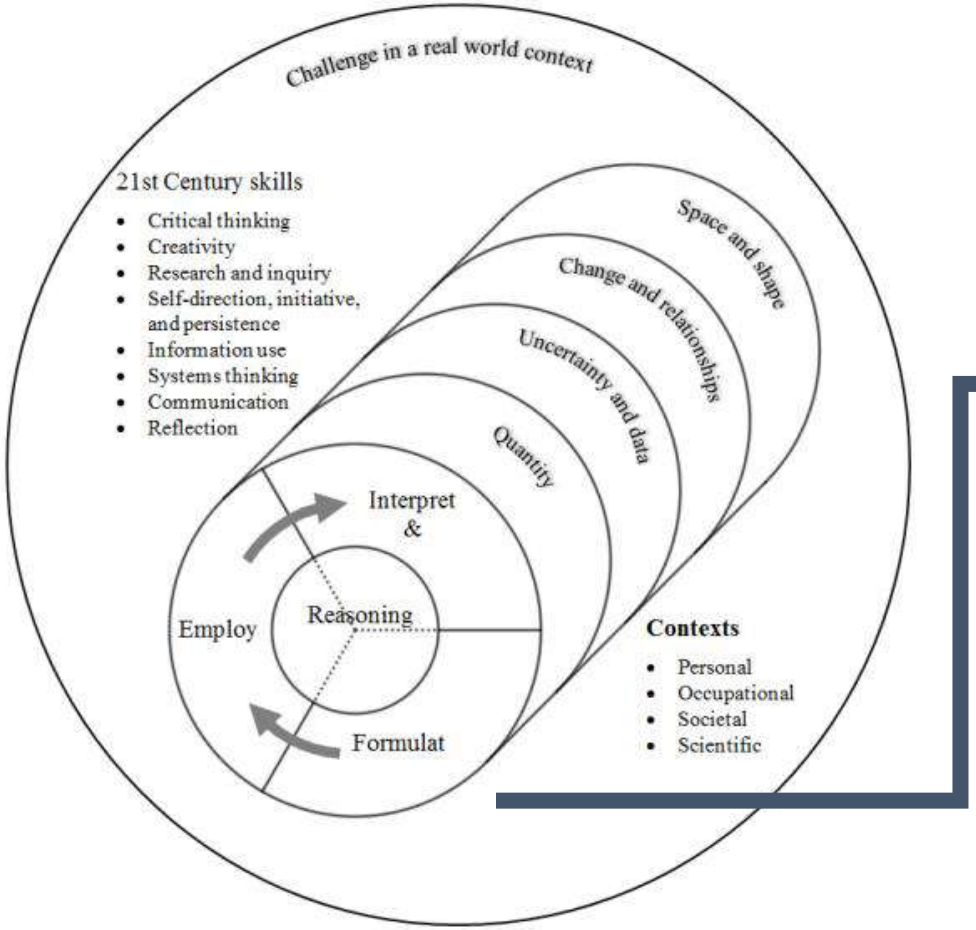
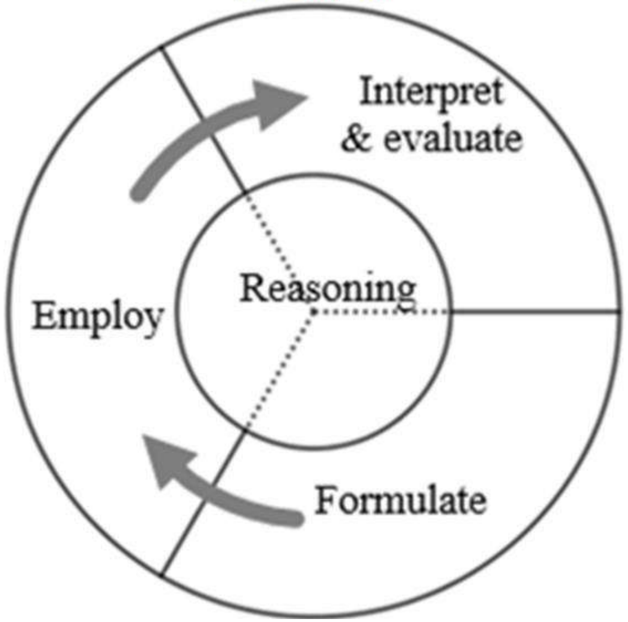


Figure 2.1. Mathematical literacy: the relationship between mathematical reasoning and the problem solving (modelling) cycle



Definition of Mathematical Literacy – cont.

Process knowledge

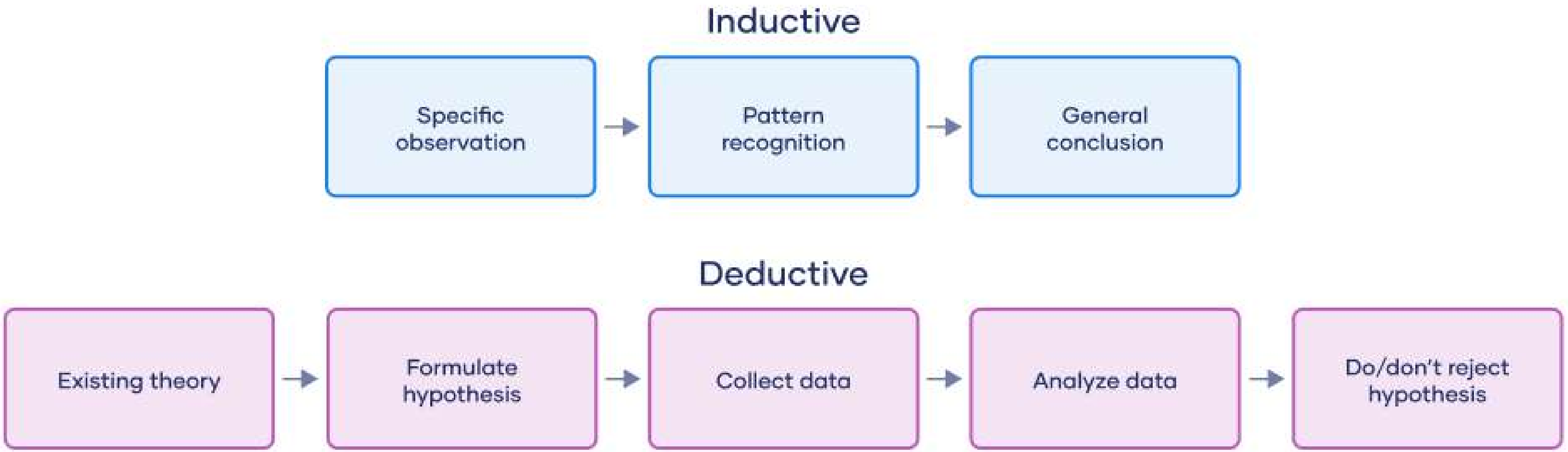
- **Formulating** situations mathematically involves applying mathematical reasoning in identifying opportunities to apply and use mathematics
- **Employing** mathematics involves applying mathematical reasoning while using mathematical concepts, procedures, facts and tools to derive a mathematical solution
- **Interpreting** mathematics involves reflecting upon mathematical solutions or results and interpreting them in the context of a problem or challenge. It involves applying mathematical reasoning to evaluate mathematical solutions in relation to the context of the problem and determining whether the results are reasonable and make sense in the situation; determining also what to highlight when explaining the solution

Process Knowledge – Mathematical Reasoning

Mathematical reasoning (both deductive and inductive) involves evaluating situations, selecting strategies, drawing logical conclusions, developing and describing solutions, and recognising how those solutions can be applied. Students reason mathematically when they:

- identify, recognise, organise, connect, and represent;
- construct, abstract, evaluate, deduce, justify, explain, and defend;
- interpret, make judgements, critique, refute, and qualify

Inductive vs. deductive reasoning



Inductive and Deductive Reasoning in Mathematics

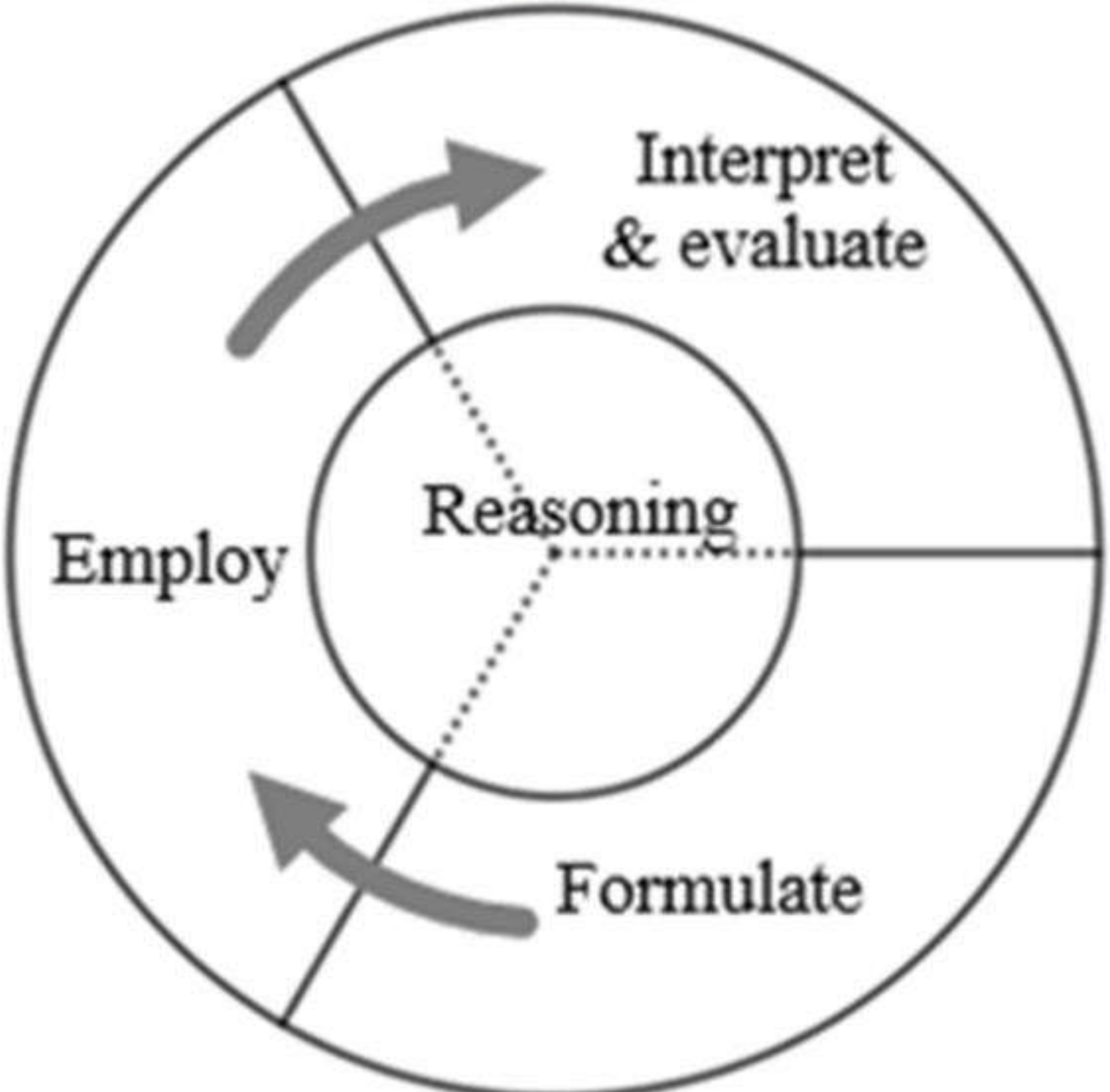
- https://www.youtube.com/watch?v=1MeN-OI6-tM

What is your classroom?

Let’s go to padlet

- Inductive → STEAM education, “sense of mathematics”, more real-life problems, more geometry focus,…
- Reductive → mathematics proof, fluency, accuracy, more number and algebra focus, …..

Figure 2.1. **Mathematical literacy: the relationship between mathematical reasoning and the problem solving (modelling) cycle**



Lets go to Padlet

Formulating situations mathematically

- selecting an appropriate model from a list;
- identifying the mathematical aspects of a problem situated in a real-world context
- identifying the significant variables;
- recognising mathematical structure (including regularities, relationships, and patterns) in problems or situations;
- simplifying a situation or problem in order to make it amenable to mathematical analysis (for example by decomposing);
- identifying constraints and assumptions behind any mathematical modelling and simplifications gleaned from the context;
- representing a situation mathematically, using appropriate variables, symbols, diagrams, and standard models;

Formulating situations mathematically – cont.

- representing a problem in a different way, including organising it according to mathematical concepts and making appropriate assumptions;
- understanding and explaining the relationships between the context-specific language of a problem and the symbolic and formal language needed to represent it mathematically;
- translating a problem into mathematical language or a representation;
- recognising aspects of a problem that correspond with known problems or mathematical concepts, facts or procedures;
- choosing among an array of and employing the most effective computing tool to portray a mathematical relationship inherent in a contextualised problem;
- creating an ordered series of (step-by-step) instructions for solving problems.

Employing mathematical concepts, facts and procedures

- performing a simple calculation;
- drawing a simple conclusion;
- selecting an appropriate strategy from a list;
- devising and implementing strategies for finding mathematical solutions;
- using mathematical tools, including technology, to help find exact or approximate solutions;
- applying mathematical facts, rules, algorithms, and structures when finding solutions;
- manipulating numbers, graphical and statistical data and information, algebraic

Employing mathematical concepts, facts and procedures – cont.

- making mathematical diagrams, graphs, simulations, and constructions and extracting mathematical information from them;
- using and switching between different representations in the process of finding solutions;
- making generalisations and conjectures based on the results of applying mathematical procedures to find solutions;
- reflecting on mathematical arguments and explaining and justifying mathematical results;
- evaluating the significance of observed (or proposed) patterns and regularities in data expressions and equations, and geometric representations;

Interpreting, applying and evaluating mathematical outcomes

- interpreting information presented in graphical form and/or diagrams;
- evaluating a mathematical outcome in terms of the context;
- interpreting a mathematical result back into the real-world context;
- evaluating the reasonableness of a mathematical solution in the context of a real-world problem;
- understanding how the real world impacts the outcomes and calculations of a mathematical procedure or model in order to make contextual judgements about how the results should be adjusted or applied;

Interpreting, applying and evaluating mathematical outcomes – cont.

- explaining why a mathematical result or conclusion does, or does not, make sense given the context of a problem;
- understanding the extent and limits of mathematical concepts and mathematical solutions;
- critiquing and identifying the limits of the model used to solve a problem;
- using mathematical thinking and computational thinking to make predictions, to provide evidence for arguments, to test and compare proposed solutions.

Definition of mathematical literacy – cont.

- Content knowledge
  - Change and relationships
  - Space and shape
  - Quantity
  - Uncertainty and data

Content topics

- Growth phenomena
- Geometric approximation
- Computer simulations
- Conditional decision making
- Functions
- Algebraic expressions
- Equations and inequalities
- Co-ordinate systems
- Relationships within and among geometrical objects in two and three dimensions
- Measurement
- Numbers and units
- Arithmetic operations:
- Percents, ratios and proportions
- Counting principles
- Estimation
- Data collection, representation and interpretation
- Data variability and its description
- Samples and sampling
- Chance and probability

- **Computer simulations**: Exploring situations (that may include budgeting, planning, population distribution, disease spread, experimental probability, reaction time modelling etc.) in terms of the variables and the impact that these have on the outcome.

Contexts

An important aspect of mathematical literacy is that mathematics is used to solve a problem set in a context. The context is the aspect of an individual’s world in which the problems are placed.

- Personal
  - focus on activities of one’s self, one’s family or one’s peer group
- Occupational
  - centred on the world of work
- Societal
  - focus on one’s community (whether local, national or global)
- Scientific
  - relate to the application of mathematics to the natural world and issues and topics related to science and technology

Item distributions

Table 2.1. **Approximate distribution of score points by domain for PISA 2022**		
		% of score points in PISA 2022
Mathematical reasoning		25
Mathematical problem solving	Formulating situations mathematically	25
	Employing mathematical concepts, facts and procedures	25
	Interpreting, applying and evaluating mathematical outcomes	25

Table 2.2. Approximate distribution of score points by content category for PISA 2022	
Content category	% of score points in PISA 2022
Change and relationships	25
Space and shape	25
Quantity	25
Uncertainty and data	25

PISA 2022 results

Performance of HK student

Year	HK Score（HK Ranking/No. of participating regions
2000+	560 (1st / 43)
2003	550 (1st / 41)
2006	547 (3rd / 57)
2009	555 (3rd / 65)
2012	561 (3rd / 65)
2015	548 (2nd / 72)
2018	551 (4th / 79)
2022	540 (4th / 81)

2022 Mathematical Performance

Ranking	Countries/Economies	Score
1	Singapore	575
2	Macao (China)	552
3	Chinese Taipei	547
4	Hong Kong (China)	540
5	Japan	536
6	Korea	527
7	Estonia	510
8	Switzerland	508
9	Canada	497
10	Netherlands	493

Level of Proficiency in PISA 2022 Mathematics

Level	What student can typically do
2	A t Level 2, students can recognise situations where they need to design simple strategies to solve problems, including running straightforward simulations involving one variable as part of their solution strategy. They can extract relevant information from oneor more sources that use slightly more complex modes of representation, such as two-way tables, charts, or two-dimensional representations of three-dimensional objects. Students at this level demonstrate a basic understanding of functional relationships and can solve problems involving simple ratios. They are capable of making literal interpretations of results .
1	At Level 1, students can answer questions involving simple contexts where all information needed is present, and the questions are clearly defined. Information may be presented in a variety of simple formats and students may need t o work with two sources simultaneously t o extract relevant information. They are able t o carry out simple, routine procedures according t o direct instructions in explicit situations, which may sometimes require multiple iterations of a routine procedure to solve a problem. They can perform actions that are obvious or that require very minimal synthesis of information, but in all instances the actions follow clearly from the given stimuli. Students at this level can employ basic algorithms, formulae, procedures, or conventions t o solve problems that most often involve whole numbers.

Level of Proficiency in PISA 2022 Mathematics

Level	What student can typically do
4	At Level 4, students can work effectively with explicit models for complex concrete situations, sometimes involving two variables, as well as demonstrate an ability to work with undefined models that they derive using a more sophisticated computational-thinking approach. Students at this level begin t o engage with aspects of critical thinking, such as evaluating the reasonableness of a result by making qualitative judgements when computations are not possible from the given information. They can select and integrate dif ferent representations of information, including symbolic or graphical, linking them directly to aspects of real-world situations. At this level, students can also construct and communicate explanations and arguments based on their interpretations, reasoning, and methodology .
3	A t Level 3, students can devise solution strategies, including strategies that require sequential decision-making or flexibility in understanding of familiar concepts. At this level, students begin using computational-thinking skills to develop their solution strategy. They are able t o solve tasks that require performing several different but routine calculations that are not all clearly defined in the problem statement. They can use spatial visualisation as part of a solution strategy or determine how to use a simulation t o gather data appropriate for the task. Students at this level can interpret and use representations based ondif ferent information sources and reason directly from them, including conditional decision-making using a two-way table. They typically show some ability t o handle percentages, fractions and decimal numbers, and to work with proportional

Level of Proficiency in PISA 2022 Mathematics

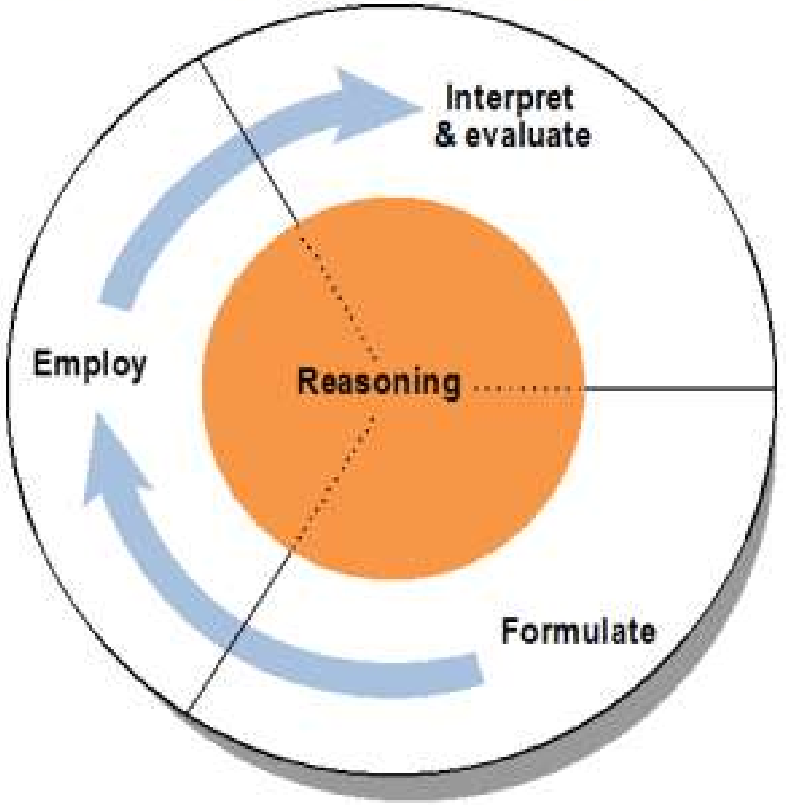
Level	What student can typically do
6	At Level 6, students can work through abstract problems and demonstrate creativity and flexible thinking to develop solutions. For example, they can recognise when a procedure that is not specified in a task can be applied in a non-standard context or when demonstrating a deeper understanding of a mathematical concept is necessary as part of a justification. They can link different information sources and representations, including effectively using simulations or spreadsheets as part of their solution. Students at this level are capable of critical thinking and have a mastery of symbolic and formal mathematical operations and relationships that they use to clearly communicate their reasoning. They can reflect on the appropriateness of their actions with respect to their solution and the original situation.
5	A t Level 5, students can develop and work with models for complex situations, identifying or imposing constraints, and specifying assumptions. They can apply systematic, well-planned problem-solving strategies for dealing with more challenging tasks, such as deciding how t o develop an experiment, designing an optimal procedure, or working with more complex visualisations that are not given in the task. Students demonstrate an increased ability t o solve problems whose solutions often require incorporating mathematical knowledge that is not explicitly stated in the task. Students at this level reflect on their work and consider mathematical results with respect to the real - world context .

Mathematical processes

- Mathematical reasoning
- Formulating situations
- Employing mathematical concepts
- Interpreting, applying, and evaluating mathematical outcomes

Figure I.2.9. The mathematical modelling cycle in PISA 2022

Mathematical processes students go through to solve real-life problems and situations



Source: *PISA 2022 Assessment and Analytical Framework* (OECD, 2023[20]).

Comparing countries and economies on the mathematics-process subscales

	Mean performance in mathematics (overall mathematics scale)	Mean performance on each mathematics-process subscale			
		Formulating	Employing	Interpreting	Reasoning
Singapore	575	576	580	577	572
Macao (China)	552	556	552	550	553
Chinese Taipei	547	550	550	548	547
Hong Kong (China)	540	542	547	540	538
Japan	536	536	536	544	534
Korea	527	526	523	531	528
Estonia	510	507	513	511	509
Switzerland	508	507	508	506	513
Canada*	497	494	495	503	499
Netherlands*	493	492	499	496	490

Math benchmarks – 2022

Country/Region	Rank	Mean	Level 1 or below (%)	Level 2 (%)	Level 3 (%)	Level 4 (%)	Level 5 (%)	Level 6 (%)
Singapore	1	575	8.0	11.2	17.6	22.6	22.0	18.6
Macao (China)	2	552	8.4	14.4	23.2	25.4	18.4	10.2
Chinese Taipei	3	547	14.6	13.5	18.7	21.5	18.0	13.7
Hong Kong (China)	4	540	13.8	14.8	21.0	23.1	16.7	10.6
Japan	5	536	12.0	16.0	24.0	25.1	16.2	6.8
Korea	6	527	16.2	16.7	22.0	22.2	14.4	8.5
Estonia	7	510	15.0	23.3	27.3	21.3	9.9	3.2
Switzerland	8	508	19.5	20.5	23.5	20.4	11.9	4.2
Canada	9	497	21.6	22.7	24.8	18.5	9.1	3.3
Netherlands	10	493	27.4	18.2	19.8	19.2	11.7	3.7

Percentage of students at each proficiency level on the mathematics process subscale: reasoning – 2022

Country/Region	Rank	Level 1 or below (%)	Level 2 (%)	Level 3 (%)	Level 4 (%)	Level 5 (%)	Level 6 (%)
Singapore	1	8.8	11.6	17.5	22.1	21.8	18.2
Macao (China)	2	8.3	14.4	22.8	25.3	18.3	10.8
Chinese Taipei	3	14.2	14.4	18.7	21.5	17.6	13.7
Hong Kong (China)	4	13.9	15.6	21.2	22.6	16.6	10.1
Japan	5	11.5	16.6	24.6	25.4	15.9	6
Korea	6	17.2	16	20.8	21.1	14.9	9.9
Estonia	7	14.9	23.1	28	21.3	9.9	2.8
Switzerland	8	17.8	19.8	23.6	21.6	12.6	4.5
Canada	9	21.9	21.6	23.6	18.5	9.9	4.5
Netherlands	10	27.2	19	20.9	19.4	10.6	2.9

Percentage of students at each proficiency level on the mathematics process subscale: formulating – 2022

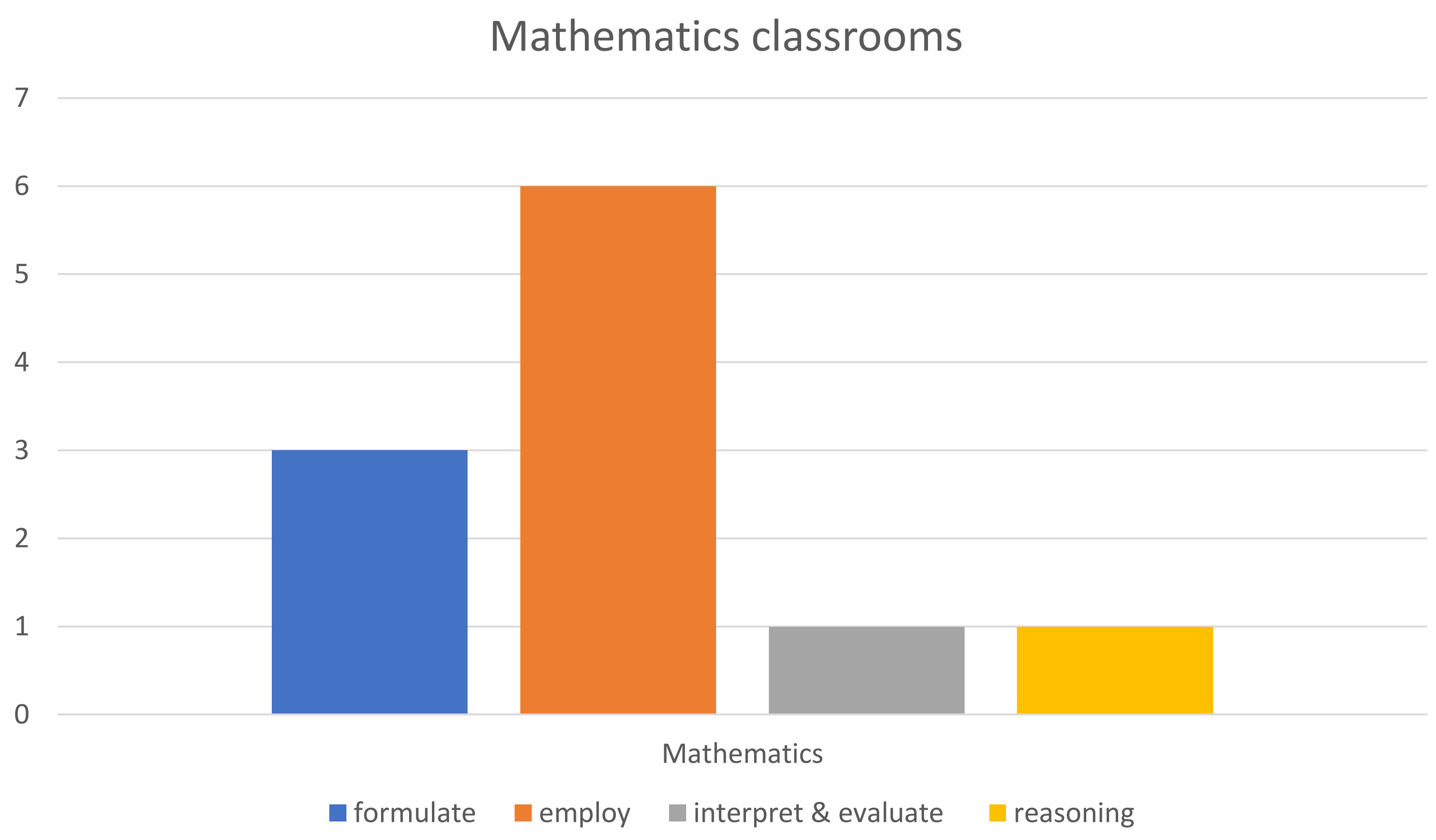
Country/Region	Rank	Level 1 or below (%)	Level 2 (%)	Level 3 (%)	Level 4 (%)	Level 5 (%)	Level 6 (%)
Singapore	1	9.2	11.4	17	21.1	20.5	20.9
Macao (China)	2	9.8	13.7	20.9	23.7	18.4	13.5
Chinese Taipei	3	14.7	13.8	18.5	19.8	17.1	16
Hong Kong (China)	4	14.5	15.1	20.2	21.5	16.3	12.3
Japan	5	13.6	15.5	22.2	23.5	16.3	8.9
Korea	6	18.3	16.1	20.9	20.3	14.1	10.4
Estonia	7	19.5	21.3	24.3	19.3	10.5	5.2
Switzerland	8	22	19.2	21.6	19	11.8	6.3
Canada	9	26	20.6	21.4	16.6	9.4	6.1
Netherlands	10	28.6	18.8	19	17.2	10.9	5.4

Percentage of students at each proficiency level on the mathematics process subscale: employing – 2022

Country/Region	Rank	Level 1 and below (%)	Level 2 (%)	Level 3 (%)	Level 4 (%)	Level 5 (%)	Level 6 (%)
Singapore	1	8.6	10.8	16.7	21.1	20.8	21.9
Macao (China)	2	8.8	14.1	23.4	25.4	17.6	10.7
Chinese Taipei	3	15	13.1	18	20	17.9	15.9
Hong Kong (China)	4	14.2	14	19.4	21.5	17	13.9
Japan	5	13.5	15.9	22.7	22.7	16.1	9
Korea	6	17.7	16.6	22.1	21.4	13.7	8.5
Estonia	7	15.5	22	26.4	21	11	4.2
Switzerland	8	20.5	20	22.5	19.3	11.9	5.7
Canada	9	24.9	21.3	22.2	16.8	9.4	5.5
Netherlands	10	28.1	15.8	17.7	17.7	12.9	7.6

Percentage of students at each proficiency level on the mathematics process subscale: interpreting – 2022

Country/Region	Rank	Level 1 and below (%)	Level 2 (%)	Level 3 (%)	Level 4 (%)	Level 5 (%)	Level 6 (%)
Singapore	1	9	11.3	17	21	20.2	21.5
Macao (China)	2	9.9	14.5	22.5	23.8	17.9	11.3
Chinese Taipei	3	15.7	13.5	18.1	20	16.7	16
Hong Kong (China)	4	15.8	14.6	19.4	20.7	16.3	13.1
Japan	5	11.8	14.9	21.5	24	17.5	10.3
Korea	6	16.5	16	21.1	21	14.5	10.9
Estonia	7	16.8	21.9	25.6	20.6	10.9	4.3
Switzerland	8	21	20.1	22.8	19.4	11.2	5.6
Canada	9	22.8	20.3	22.1	17.9	10.6	6.4
Netherlands	10	28.3	17.4	18.4	17.8	12	6.2



Mathematics classrooms
7
6
5
4
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1
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Mathematics
formulate
employ
interpret & evaluate
reasoning

Mathematical content

- Quantity
- Uncertainty and data
- Change and relationships
- Space and shape

Comparing countries and economies on the mathematics-content subscales

	Mean performance in mathematics (overall mathematics scale)	Mean performance on each mathematics-content subscale			
		Change and relationship	Quantity	Space and shape	Uncertainty and data
Singapore	575	574	579	571	579
Macao (China)	552	551	551	555	551
Chinese Taipei	547	549	547	551	546
Hong Kong (China)	540	536	545	540	542
Japan	536	533	535	541	540
Korea	527	525	527	537	524
Estonia	510	508	515	513	503
Switzerland	508	504	510	518	502
Canada*	497	502	494	491	500
Netherlands*	493	489	497	485	496

Percentage of students at each proficiency level on the mathematics content subscale: quantity – 2022

Country/Region	Rank	Level 1 or below (%)	Level 2 (%)	Level 3 (%)	Level 4 (%)	Level 5 (%)	Level 6 (%)
Singapore	1	8.7	10.6	16.9	21.2	20.9	21.7
Macao (China)	2	9.4	14.5	22.6	24.3	18	11.3
Chinese Taipei	3	15.7	13.6	17.8	20.6	17.3	15.1
Hong Kong (China)	4	14.6	14.1	19.3	21.7	16.8	13.5
Japan	5	14.3	15.7	22.2	22.6	15.8	9.4
Korea	6	16.9	16.1	21.8	21.8	14.8	8.7
Estonia	7	15.6	21.2	25.8	21.1	11.3	5
Switzerland	8	19.9	19.6	22.4	20.4	12.2	5.6
Canada	9	24.7	21.5	22.4	17.3	9.3	4.8
Netherlands	10	28	16.5	18.8	18.4	12	6.3

Percentage of students at each proficiency level on the mathematics content subscale: uncertainty and data – 2022

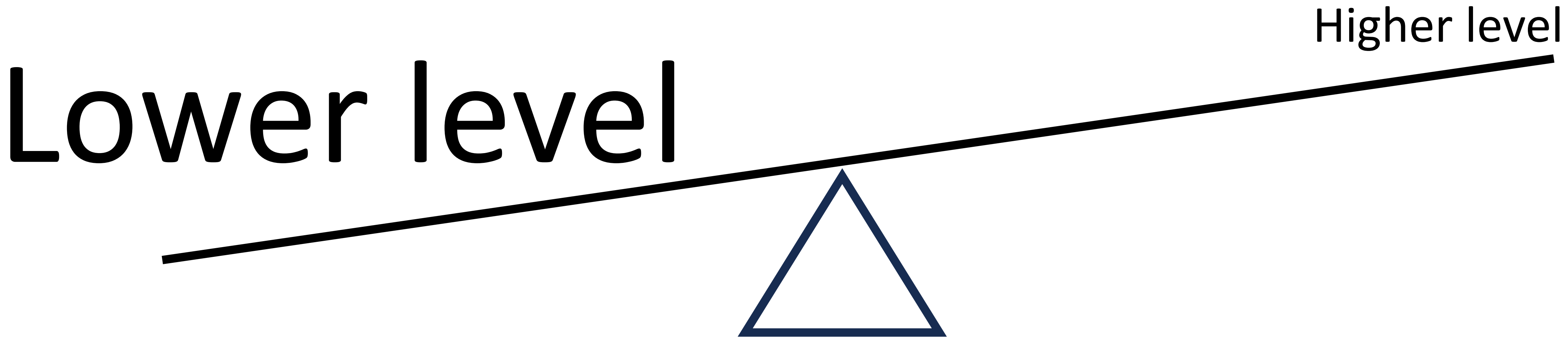
Country/Region	Rank	Level 1 and below (%)	Level 2 (%)	Level 3 (%)	Level 4 (%)	Level 5 (%)	Level 6 (%)
Singapore	1	9.4	11.1	16.5	20.4	20.2	22.3
Macao (China)	2	9.6	14.2	22.5	24.6	17.9	11.2
Chinese Taipei	3	16.1	13.5	17.8	20	17.1	15.5
Hong Kong (China)	4	15.9	14.1	18.8	20.7	16.9	13.5
Japan	5	12.4	15.4	22.5	23.7	16.4	9.6
Korea	6	18.3	16.7	21	20.2	13.9	9.9
Estonia	7	18.3	23	26.3	19.6	9.2	3.6
Switzerland	8	22.8	19.9	21.9	18.3	11.3	5.8
Canada	9	23.8	20.2	21.7	17.3	10.4	6.5
Netherlands	10	28.4	17.7	17.7	17.2	12.1	6.9

Percentage of students at each proficiency level on the mathematics content subscale: change and relationships – 2022

Country/Region	Rank	Level 1 or below (%)	Level 2 (%)	Level 3 (%)	Level 4 (%)	Level 5 (%)	Level 6 (%)
Singapore	1	8.7	11.5	17.3	22.1	21.4	19
Macao (China)	2	9.8	14.3	22.2	24.5	17.8	11.4
Chinese Taipei	3	14.8	14	18.1	20.2	17.6	15.4
Hong Kong (China)	4	14.9	15.2	21.1	22.3	16.3	10.2
Japan	5	12.9	16.2	23.5	24.8	15.5	7.1
Korea	6	18.3	16.2	21	20.2	14.5	9.8
Estonia	7	16.7	22.5	26.4	21	9.9	3.5
Switzerland	8	21.9	19.7	22.5	19.5	11.7	4.9
Canada	9	22.2	20.8	23	18.1	10.2	5.8
Netherlands	10	28.6	16.7	19.8	18.7	11.6	4.6

Percentage of students at each proficiency level on the mathematics content subscale: space and shape – 2022

Country/Region	Rank	Level 1 or below (%)	Level 2 (%)	Level 3 (%)	Level 4 (%)	Level 5 (%)	Level 6 (%)
Singapore	1	9.4	12.1	17.7	21.6	20.4	18.8
Macao (China)	2	9.1	14.2	22.3	23.8	17.5	13.1
Chinese Taipei	3	13.2	14.3	18.7	21.1	17.4	15.2
Hong Kong (China)	4	13.5	15.9	21.5	21.9	16	11.2
Japan	5	10.3	16.2	24.1	24.6	16.6	8.2
Korea	6	15.4	15.5	21.1	20.9	15.2	11.9
Estonia	7	16.3	21.8	25.3	20.3	11.3	5
Switzerland	8	17.4	19.1	22.7	20.9	13.2	6.7
Canada	9	25.8	21.2	22.2	16.7	9.1	5.1
Netherlands	10	27.9	21	21.6	16.9	9.1	3.4

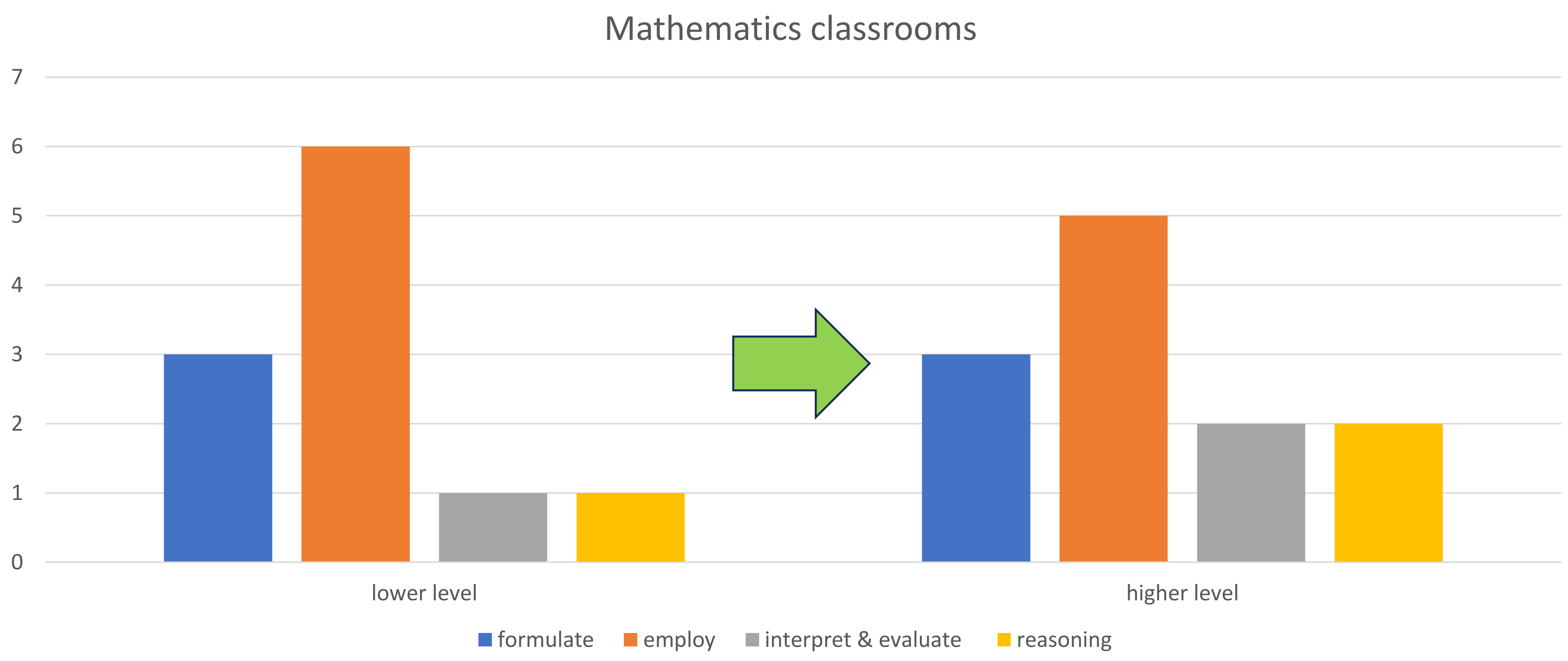

Higher level
Lower level

What do you think ? Why?

Go to Padlet and share

My sharing

- Problem formulation?
- Data handling (tables)?
- Well-structured (informed) questions?
- Reasoning = solutions?
- Contextual problems?
- Teacher beliefs on “mathematics thinking”?


Mathematics classrooms
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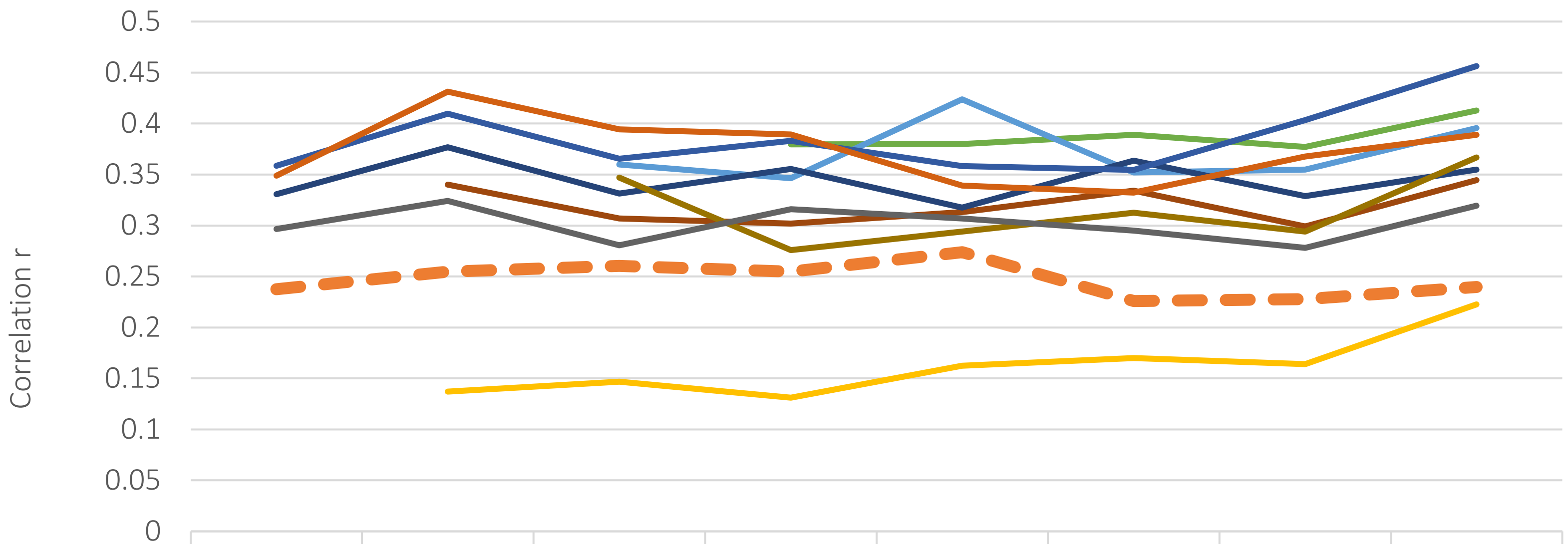
lower level
higher level
formulate
employ
interpret & evaluate
reasoning

Socio-economic Equity in Mathematics



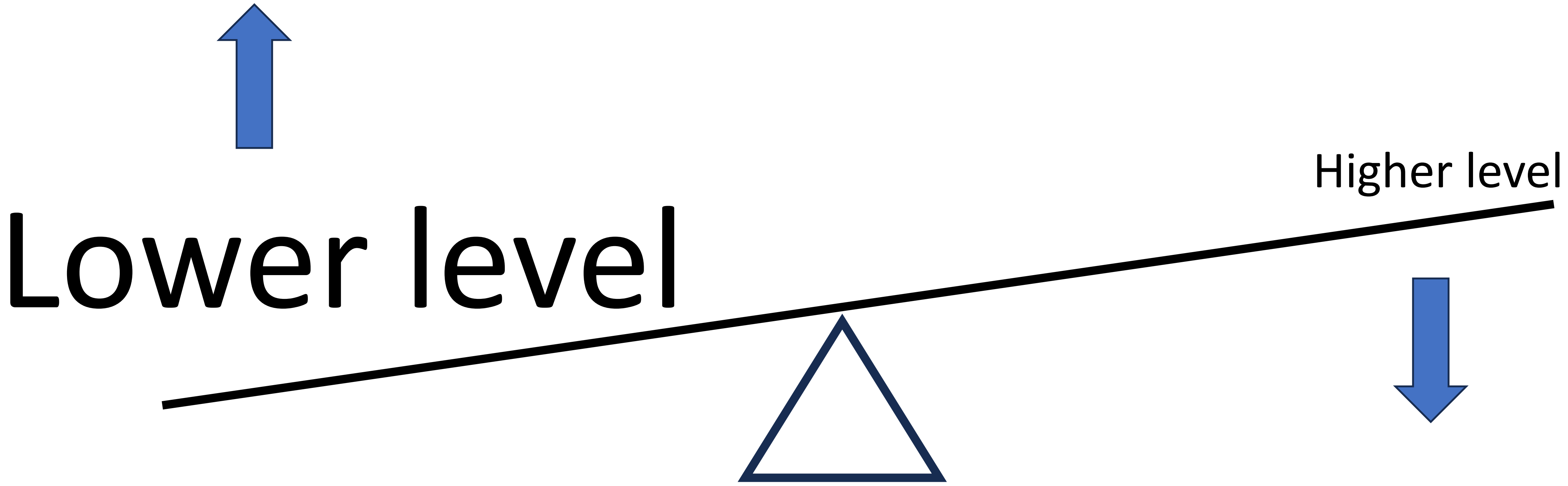
	2000	2003	2006	2009	2012	2015	2018	2022
Singapore				0.38	0.38	0.39	0.38	0.41
Macao-CN		0.14	0.15	0.13	0.16	0.17	0.16	0.22
Chinese Taipei			0.36	0.35	0.42	0.35	0.35	0.40
HK-CN	0.24	0.25	0.26	0.25	0.27	0.23	0.23	0.24
Japan		0.34	0.31	0.30	0.31	0.33	0.30	0.34
Korea	0.33	0.38	0.33	0.36	0.32	0.36	0.33	0.35
Estonia			0.35	0.28	0.29	0.31	0.29	0.37
Switzerland	0.36	0.41	0.37	0.38	0.36	0.35	0.40	0.46
Canada	0.30	0.32	0.28	0.32	0.31	0.29	0.28	0.32
Netherlands	0.35	0.43	0.39	0.39	0.34	0.33	0.37	0.39

Math performance by gender

	Female	Male
Singapore	568	581
Macao (China)	544	559
Chinese Taipei	544	550
Hong Kong (China)	536	544
Japan	531	540
Korea	525	530
Estonia	507	513
Switzerland	502	513
Canada	491	503
Netherlands	487	498

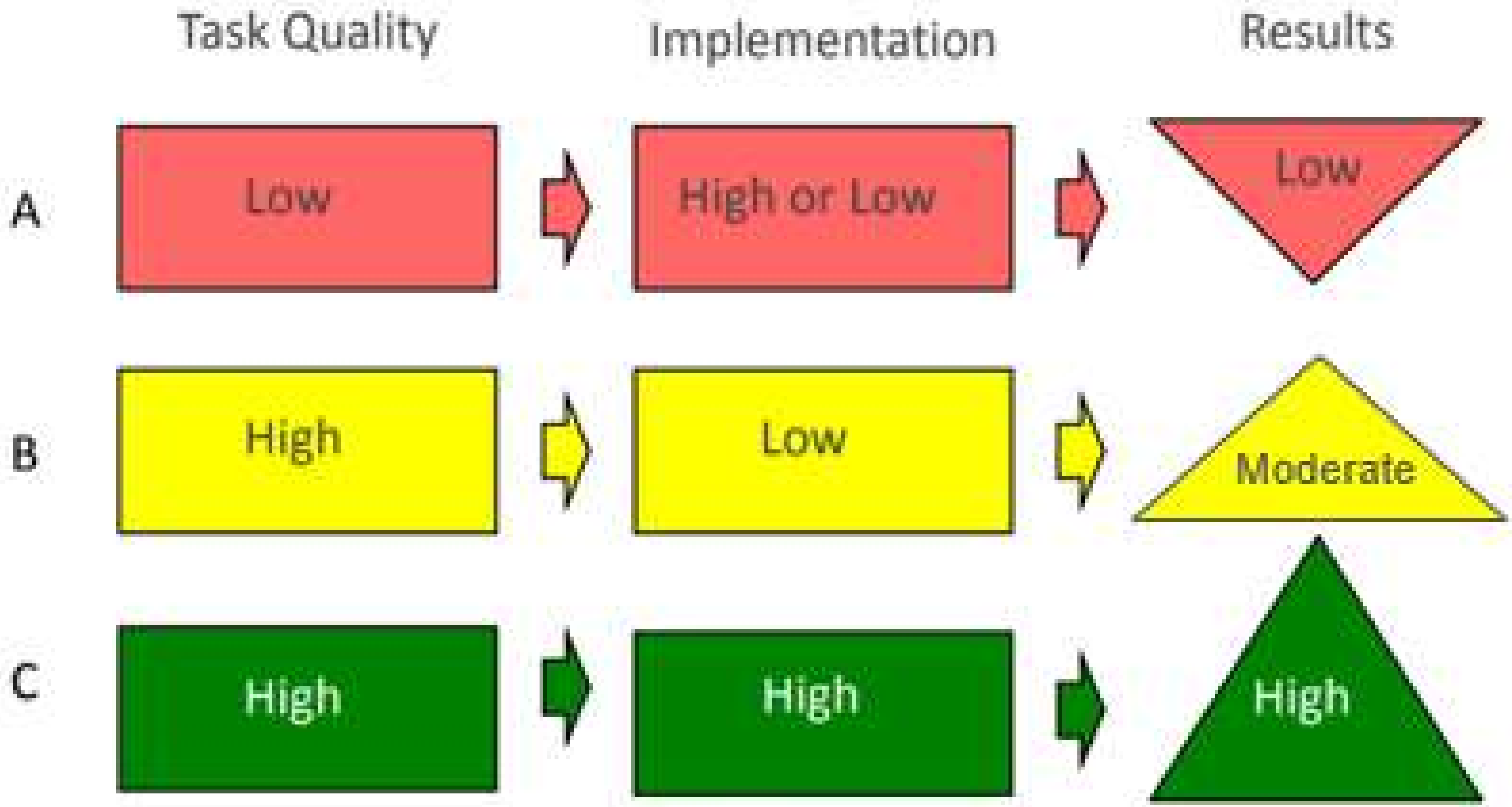
Gender Equity in Mathematics

Regions	Corelations
Singapore	0.06
Macao (China)	0.08
Chinese Tapei	0.03
Hong Kong (China)	0.04
Japan	0.05
Korea	0.02
Estonia	0.04
Switzerland	0.06


Higher level
Lower level

My sharing

- Task design
- What is
- Learning and Teaching with technology
- STEM education



Stein, M.K. & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation 2*(4), 50-80.

Ill-structured problems

- Students create their own questions
- Ask students to identify the information they need to complete the question, e.g., solve $x + b = 0$ and $x + b - a = 0$
- Rich tasks, see some examples.

Non-procedural knowledge learning

- Think tac toe

Think Tac Toe

The Pythagorean Theorem

***Directions:* Complete the activities described in either one vertical or one diagonal row.**

Draw a right triangle and label the right angle, legs, and hypotenuse. State the relationship of the sides of a triangle.	Name a career in which one would have to use the Pythagorean Theorem. Give an example of when, where and how it would be used.	Design a teaching tool with a diagram of a proof of the Pythagorean Theorem. Label it for all to understand.
Complete all of the EVEN Practice Problems on p. 266 of your Prentice Hall text.	Complete the Practice Problems found at this site: http://regentsprep/Regents/math/fpyth/PracPyth.htm	Create four (4) real world problems that would need the use of the Pythagorean Theorem. Show the solutions.
Determine a set of 8 Pythagorean "TRIPLES." Prove them with equations.	Write a descriptive essay about Pythagoras: his life, accomplishments, and failures.	Find another mathematical theorem. State it, diagram its proof, and write a paragraph about why, how and

Use a plastic square sheet to estimate the area of a regular polygon.	Use lego cubes with sides of 1 cm to build a quadrilateral prism of volume = 30 cm3.	Pour water into a cylinder to estimate the volume of water.
Put 10 bricks into a cylinder with water to estimate the volume of each brick.i.n	Put 10 brick cubes into a cylinder with water to estimate the side of the cube.	Draw all the faces seen on a prism.
Use a plastic square sheet to estimate the area of a curved shape.	Use lego rectangular prism with sides of 1x2x1 cm to build a cube.	Use a plastic square sheet to estimate the area of an irregular polygon.

Odd one out

$\sqrt{2}$	$\sqrt{4}$
$\sqrt{8}$	$\sqrt{-2}$

- ❖ $\sqrt{2}$ is a irrational number
- ❖ $\sqrt{4}$ integer
- ❖ $\sqrt{8}$ surd that is not the simplest form
- ❖ $\sqrt{-2}$ imaginary number

- 萬事起頭難，亦可以很易。

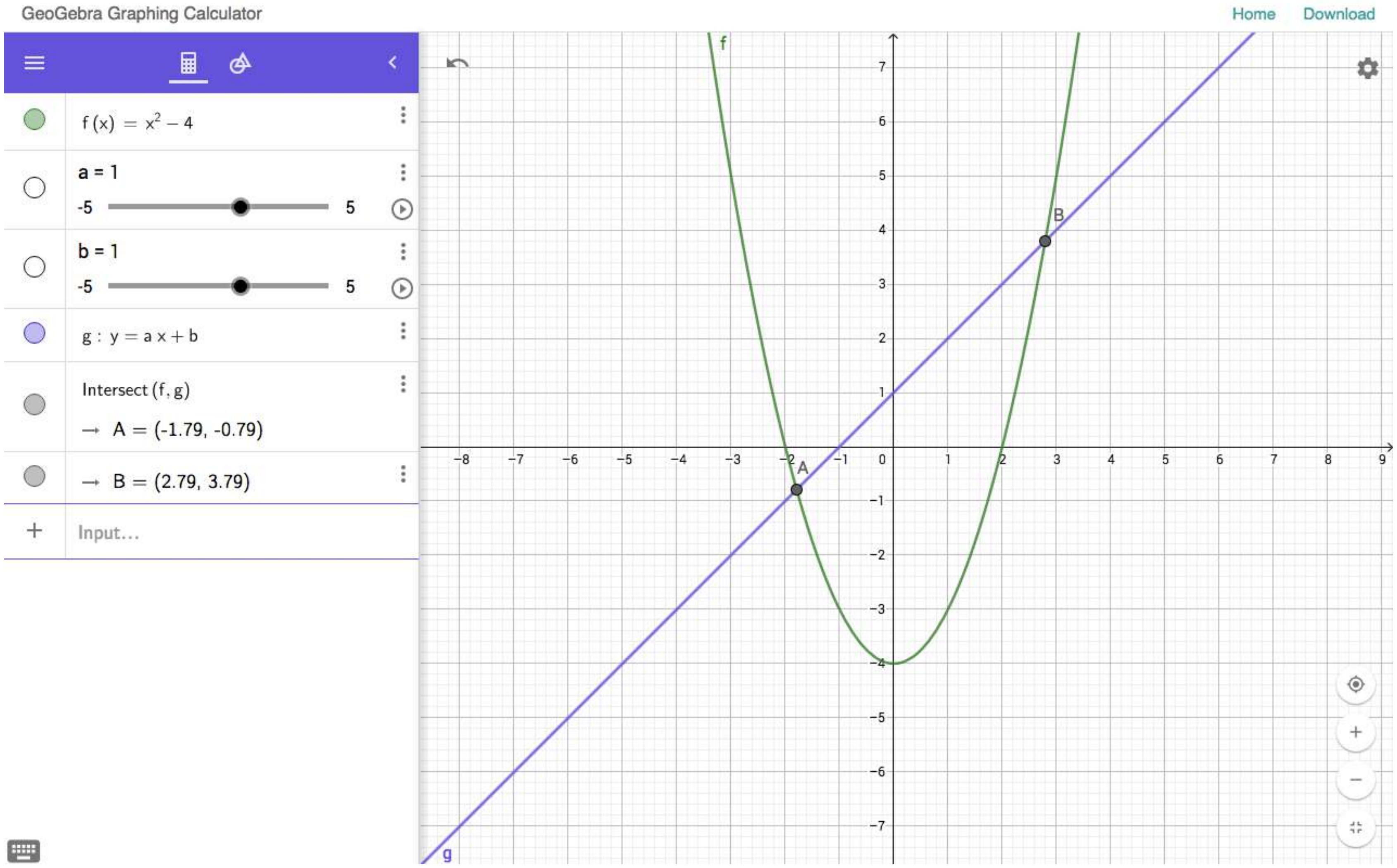
M2 - Radian

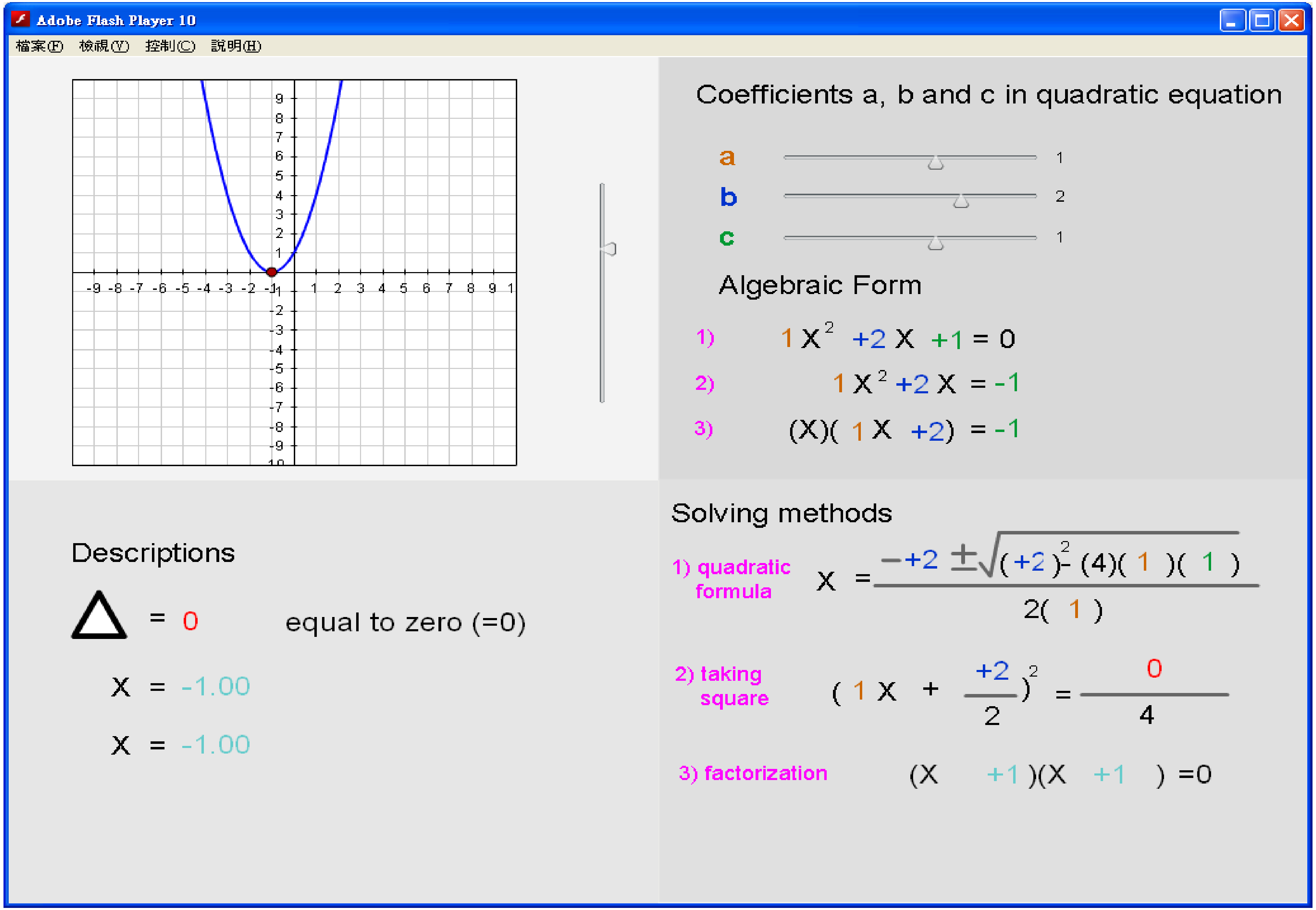
$\pi r^2 \frac{\theta}{360^\circ}$	$2\pi r \frac{\theta}{2\pi}$
$\pi r^2 \cdot \frac{\theta}{2\pi}$	$\frac{1}{2} r^2 \theta$

$\sin\theta$	$\cos(\pi+\theta)\tan(\pi+\theta)$
$\cos(270^\circ+\theta)$	$\sin(180^\circ-\theta)$

3. 教學法>工具

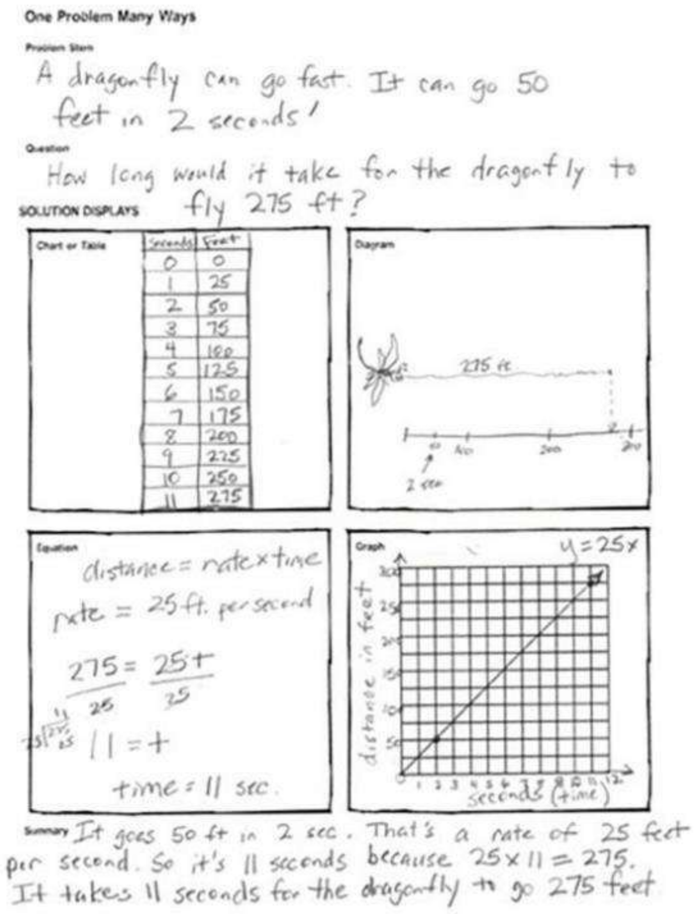
Low Quality Tasks GeoGebra



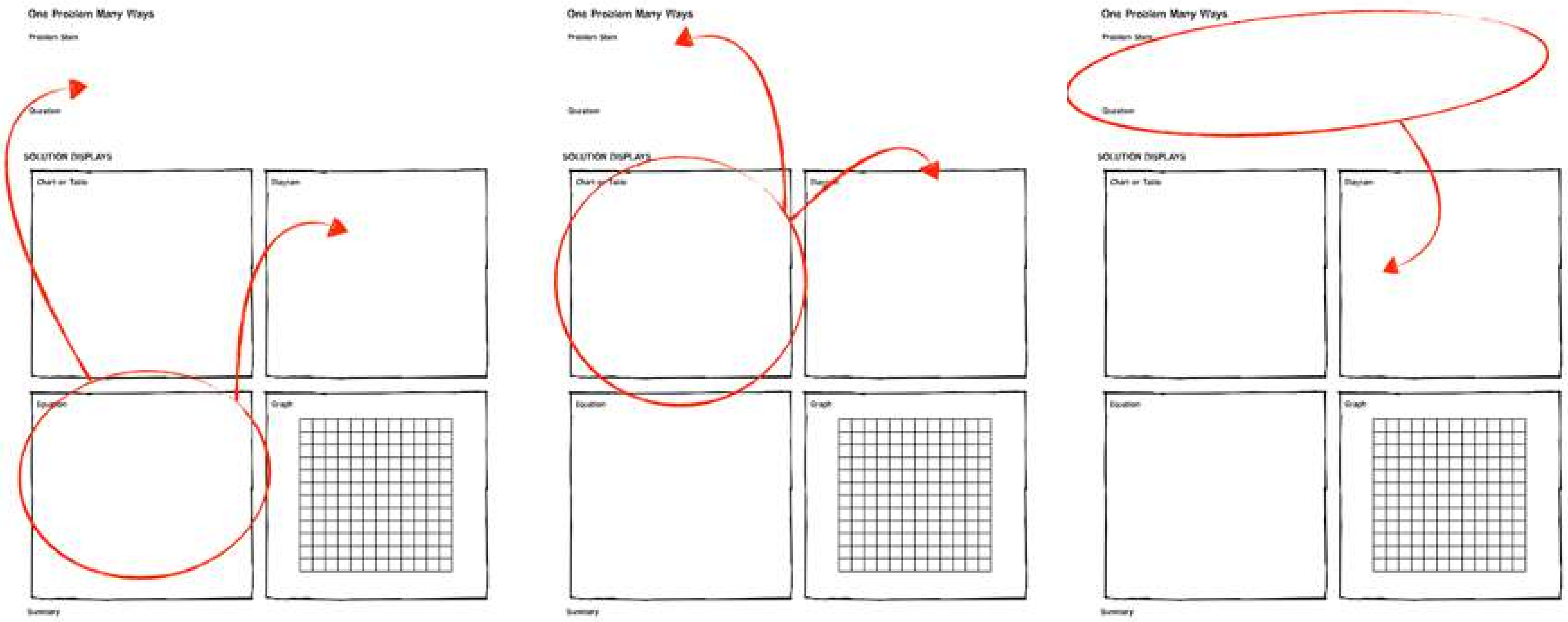

Adobe Flash Player 10
檔案(F) 檢視(V) 控制(C) 說明(H)
Coefficients a, b and c in quadratic equation
a 1
b 2
c 1
Algebraic Form
1) 1 X 2 +2 X +1 = 0
2) 1 X 2 +2 X = -1
3) (X)(1 X +2) = -1
Descriptions
= 0 equal to zero (=0)
X = -1.00
X = -1.00
Solving methods
1) quadratic formula
2) taking square
3) factorization
(X +1)(X +1) =0

Multiple Representations

dragonfly problem. Examine the various ways that this problem can be represented by student



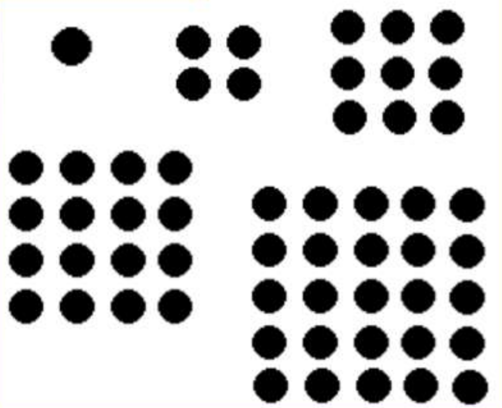
Opportunities for Reasoning



Table

n	n^{th} term
1	1
2	4
3	9
4	16
5	25
⋮	⋮

Diagram 1



Equation

$$a_n = n^2$$ ← from digram 1

$a_1 = 1$ ← from diagram 2

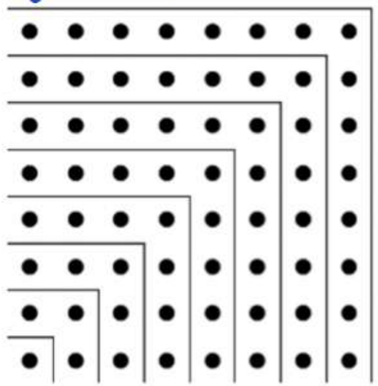
$a_2 = 1+3$

$a_3 = 1+3+5$

⋮

$a_n = 1+3+5+\dots+(2n-1)$

Diagram 2



Chapter 3 Quadratic Equations in One Unknown

Summary Table

$$x^2+3x-18=0$$

Solving quadratic equation

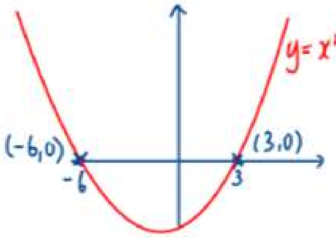
① $(x-3)(x+6)=0$

$x-3=0$ or $x+6=0$

$x=3$ or $x=-6$

② $x=\frac{-3\pm\sqrt{3^2-4(1)(-18)}}{2(1)}=3$ or -6

Graph



Discriminant Δ

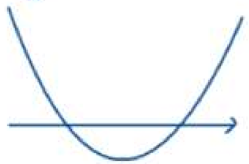
$\Delta=3^2-4(1)(-18)$

$=81>0$

$\therefore$ The eqt. has two real roots

Graph

Graph has two x-intercepts



Forming quadratic equation

Let α and β be the roots of $x^2+3x-18=0$.

Form an equation in x with roots 2α and 2β

$\alpha+\beta=-\frac{3}{1}=-3$ ✓

$\alpha\beta=\frac{-18}{1}=-18$ ✓

$2\alpha+2\beta=\ldots=-6$

$(2\alpha)(2\beta)=\ldots=-72$

The req. eqt.:

$x^2-(2\alpha+2\beta)x+(2\alpha)(2\beta)=0$

$x^2-2(\alpha+\beta)x+4\alpha\beta=0$

$x^2-2(-3)x+4(-18)=0$

$x^2+6x-72=0$

Sum and product of roots

Sum of roots $=-\frac{3}{1}=-3$

product of roots $=\frac{-18}{1}=-18$

Roots α and β

$\alpha^2+3\alpha-18=0$

$\beta^2+3\alpha-18=0$

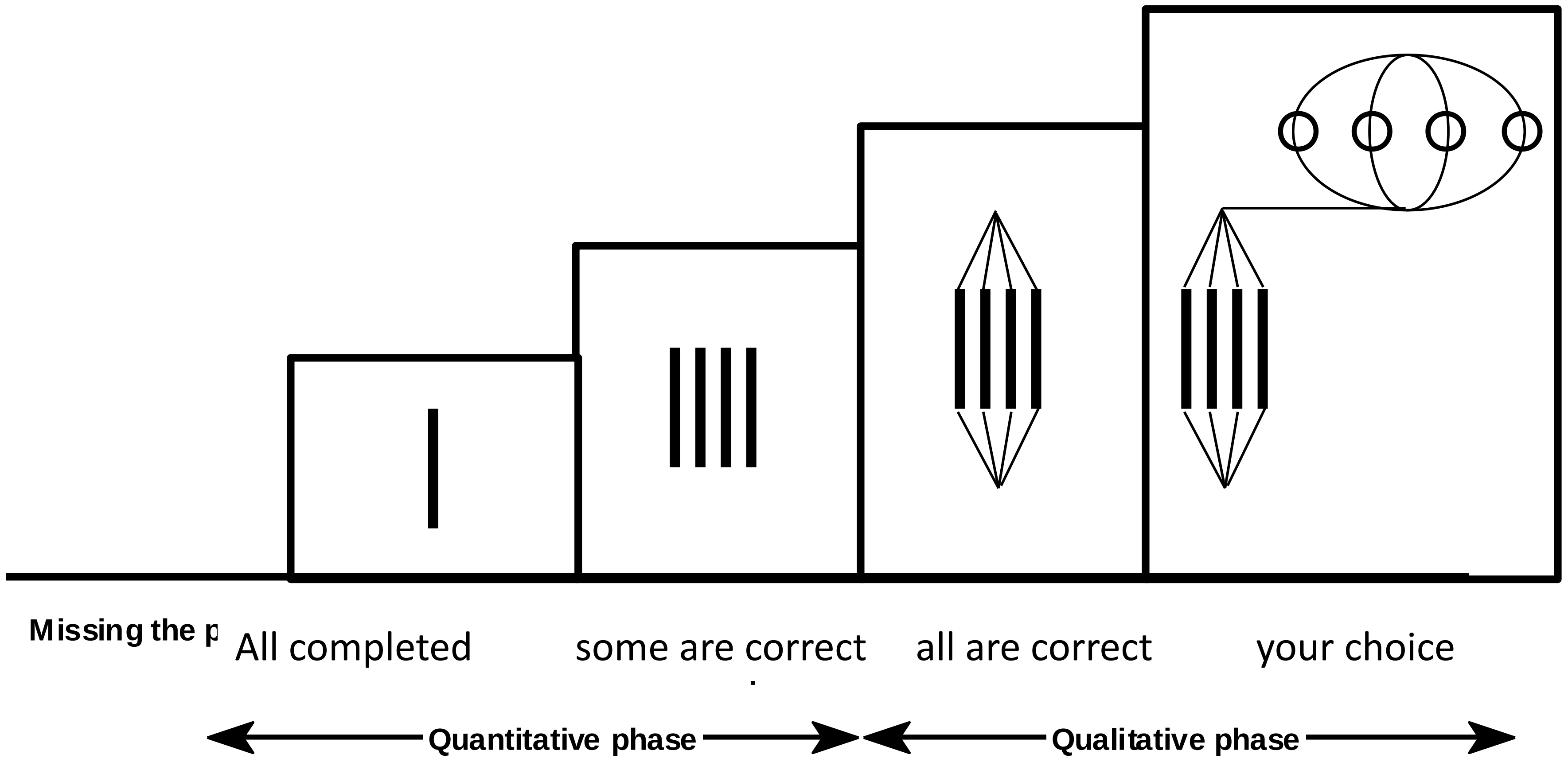
Given eqt. → OLD: sum of root $=-\frac{b}{a}$, product of root $=\frac{c}{a}$ → NEW: sum of root, product of root $\xrightarrow{\text{form}}$ new equation

$x^2-(\text{sum})x+\text{product}=0$

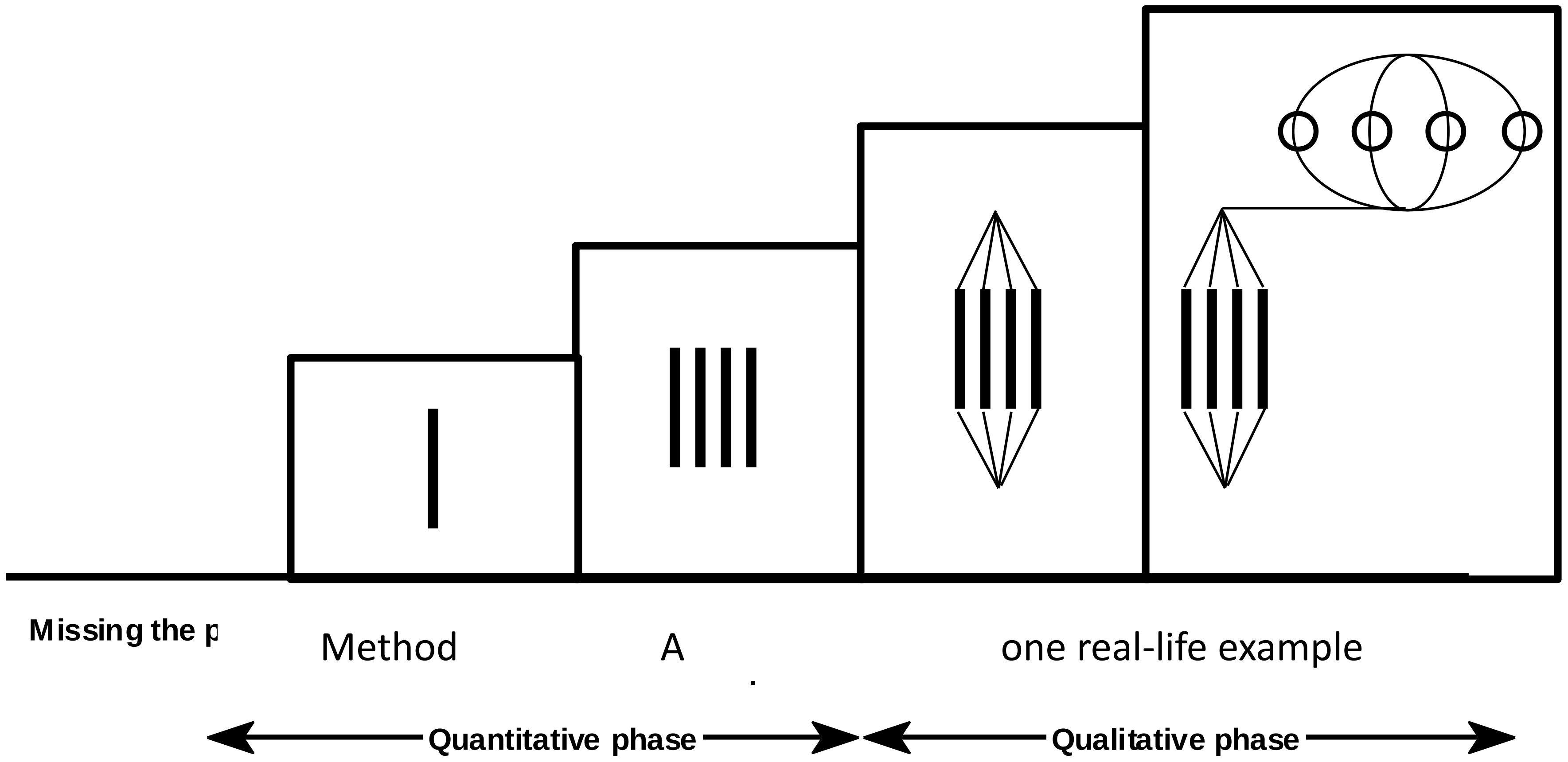
Mathematic concept development

- What is concept of triangle?
- What is concept of linear equations?

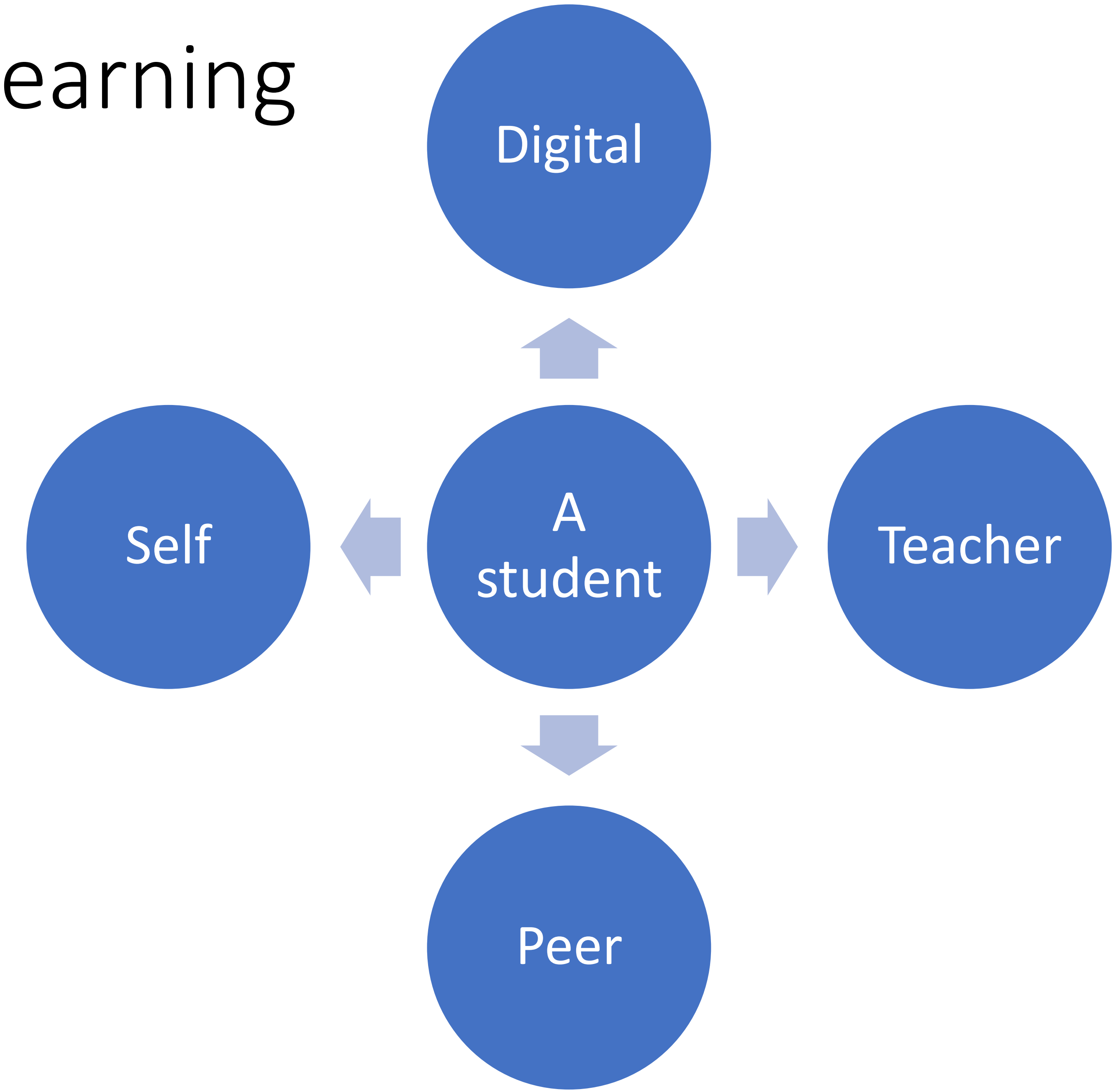
Quantitative → qualitative



Quantitative → qualitative

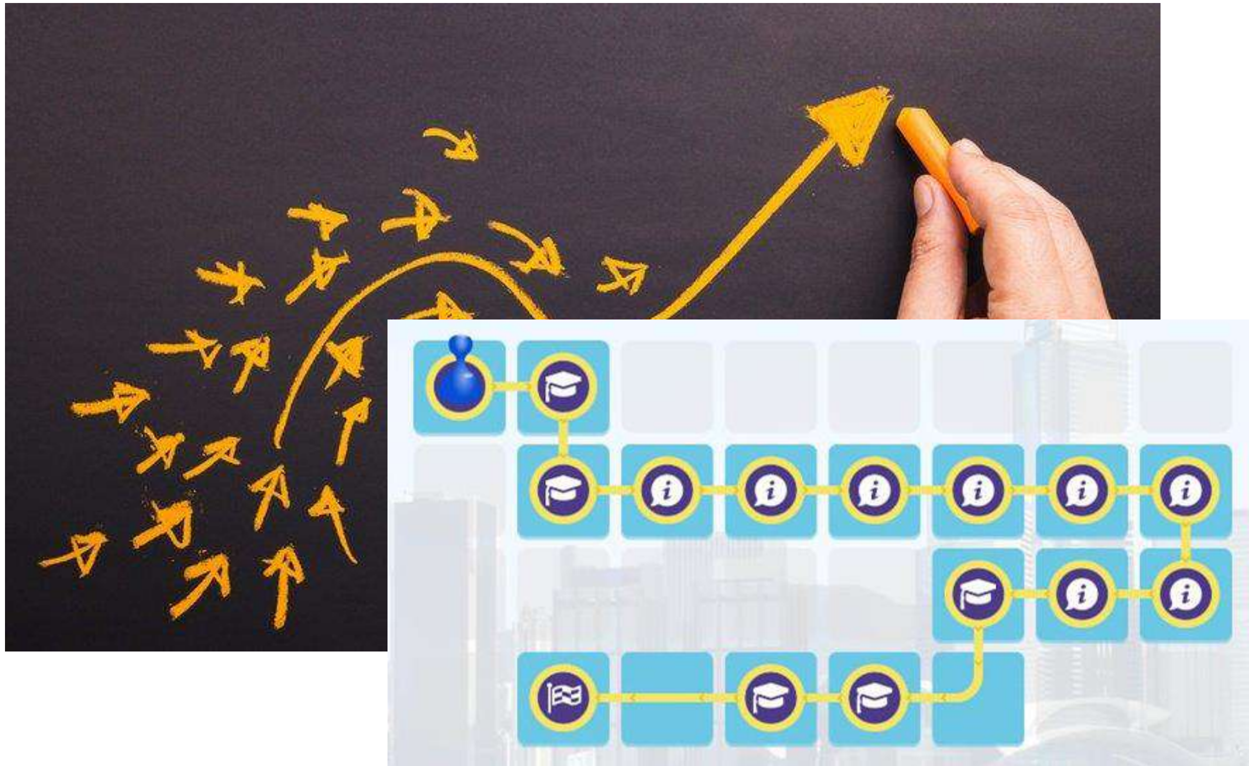


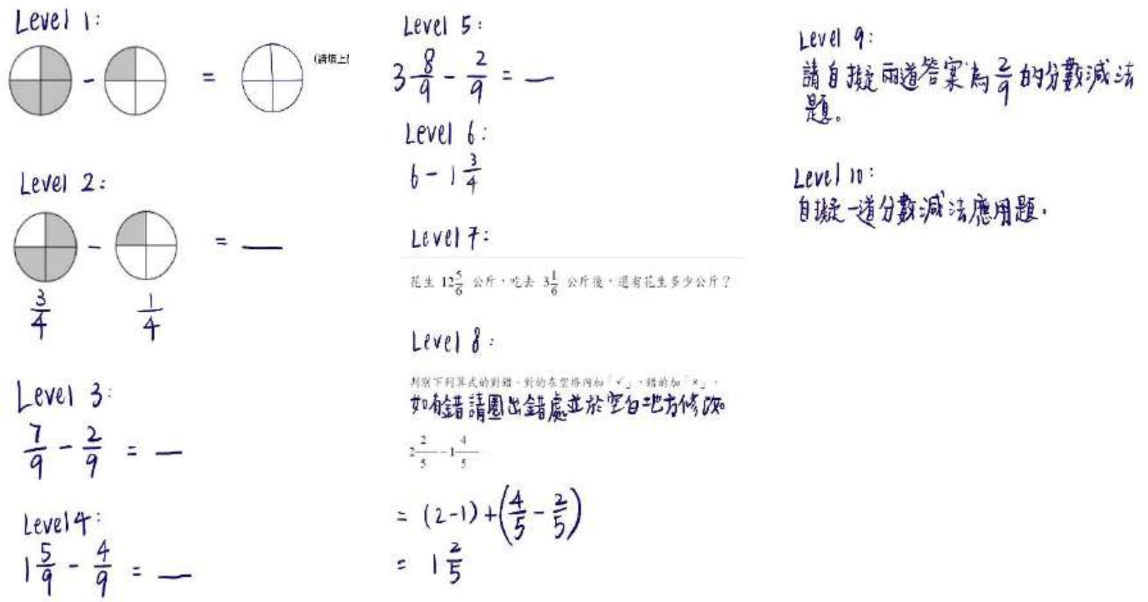
Blended learning

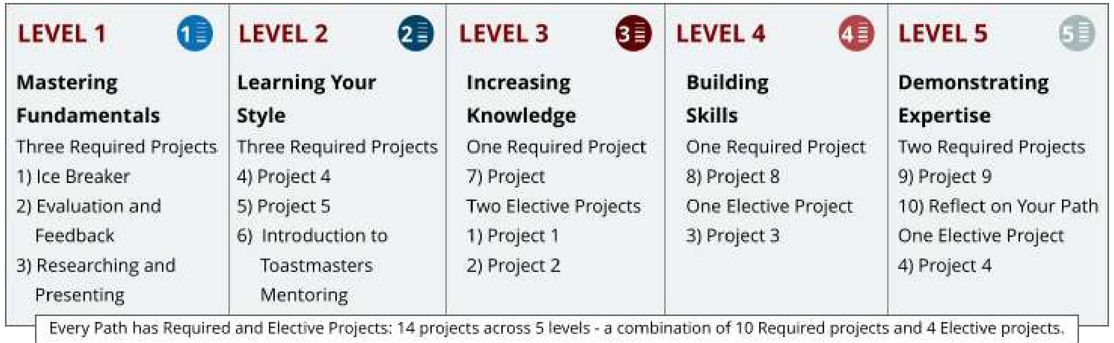


Learning path

- 從時間表/教學安排到學習路徑
- 升級練習（差異化任務)







LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5
Mastering Fundamentals	**Learning Your Style**	**Increasing Knowledge**	**Building Skills**	**Demonstrating Expertise**
Three Required Projects 1) Ice Breaker 2) Evaluation and Feedback 3) Researching and Presenting	Three Required Projects 4) Project 4 5) Project 5 6) Introduction to Toastmasters Mentoring	One Required Project 7) Project Two Elective Projects 1) Project 1 2) Project 2	One Required Project 8) Project 8 One Elective Project 3) Project 3	Two Required Projects 9) Project 9 10) Reflect on Your Path One Elective Project 4) Project 4

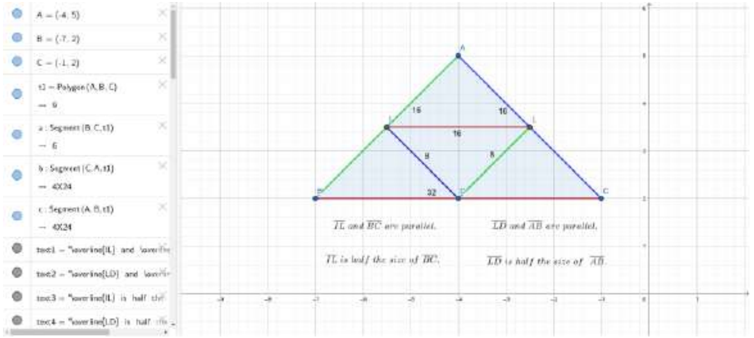
Every Path has Required and Elective Projects: 14 projects across 5 levels - a combination of 10 Required projects and 4 Elective projects.

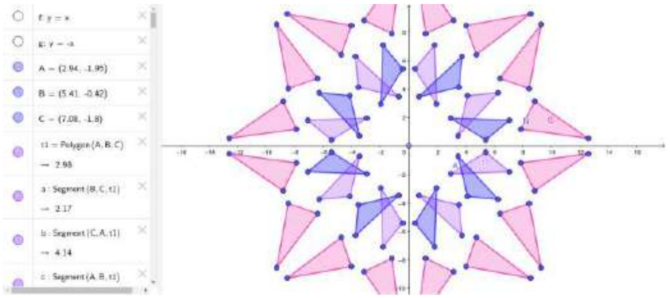
Learning Path

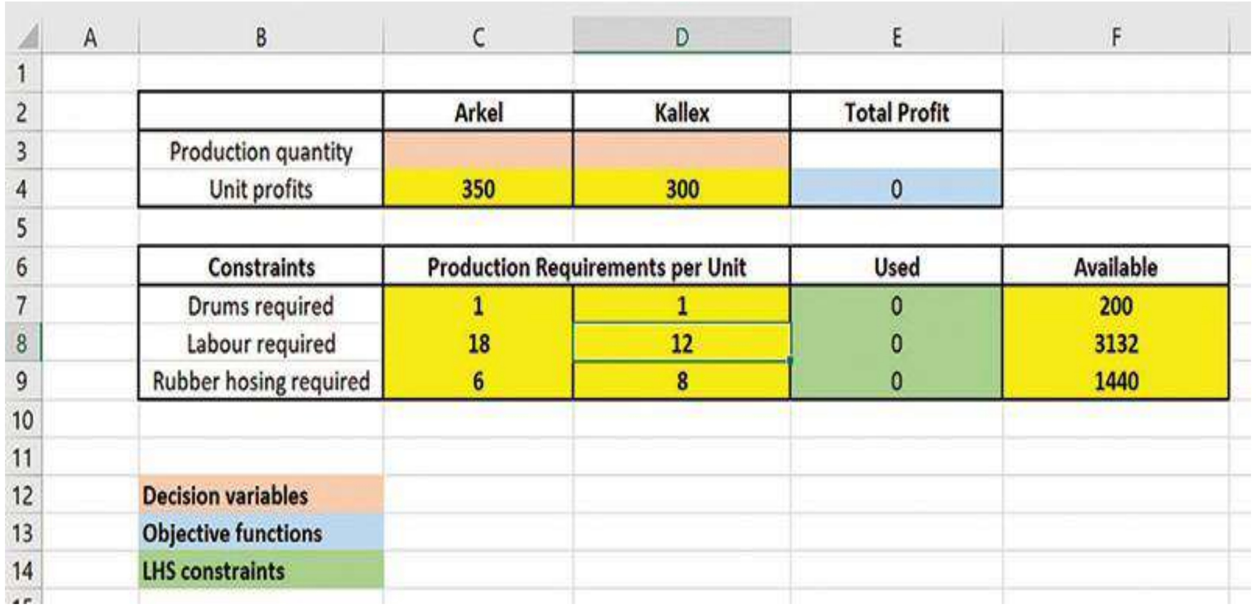
Path	Task
Formulating	Task 1, 2, 3; game 5, 6; MC 3, 5
Employing	Game 1, 2, task 4, 5; MC 1, 2
Interpreting	Task 6, 7; MC 3, 4
Evaluating	Task 1, 2, 4, 6, 7
reasoning	Game 1, 4 ,5; Task 3, 5

Teaching with technology

- Moving from manipulation and demonstration to data collection







	A	B	C	D	E	F
1						
2			Arkel	Kallex	Total Profit	
3		Production quantity				
4		Unit profits	350	300	0	
5						
6		Constraints	Production Requirements per Unit		Used	Available
7		Drums required	1	1	0	200
8		Labour required	18	12	0	3132
9		Rubber hosing required	6	8	0	1440
10						
11						
12		Decision variables				
13		Objective functions				
14		LHS constraints				

STEAM Education

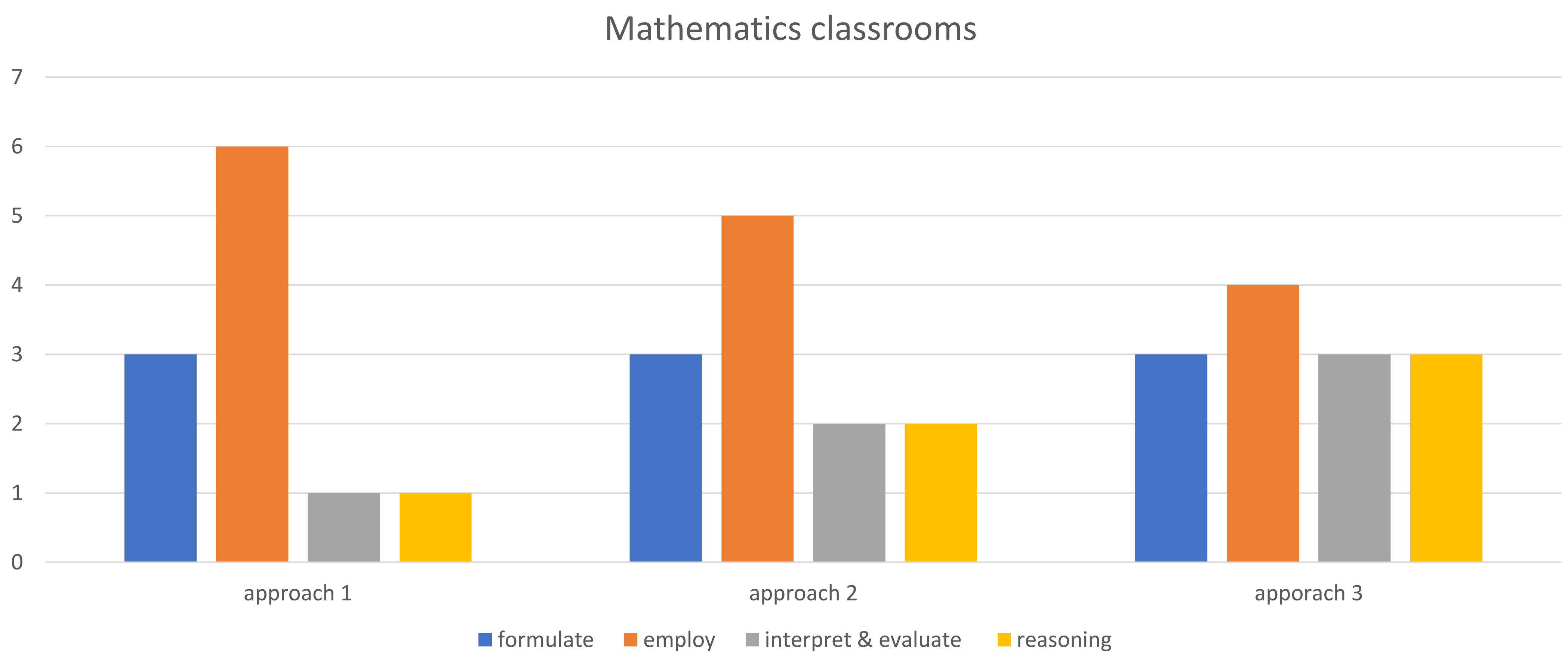
- Using Tables
- Estimation
- Prediction
- Explain the STEAM using mathematics language

Mathematics Teacher Beliefs

- Accuracy oriented
- Procedural knowledge oriented
- Mathematics “definition” oriented
- Mathematics “concept” oriented
- Rote learning
- Rule based learning
- They are all GOOD, but …….偏食
- “You are what you eat”

An Emerging topic in mathematics
Artificial intellegnece

Choices?



Mathematics classrooms
7
6
5
4
3
2
1
0
approach 1
approach 2
apporach 3
formulate
employ
interpret & evaluate
reasoning

Thank You

Thomas Chiu
Assistant Professor
Department of Curriculum
and Instruction
The Chinese University of Hong Kong

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Table I.A1.6. Proficiency levels on the mathematical content subscale: *Quantity*

Level	What students can typically do
6	At Level 6 and above, students conceptualise and work with models of complex quantitative processes and relationships; devise strategies for solving problems; formulate conclusions, arguments and precise explanations; interpret and understand complex information, and link multiple complex information sources; interpret graphical information and apply reasoning to identify, model and apply a numeric pattern. They are able to analyse and evaluate interpretive statements based on data provided; work with formal and symbolic expressions; plan and implement sequential calculations in complex and unfamiliar contexts, including working with large numbers, for example to perform a sequence of currency conversions, entering values correctly and rounding results. Students at this level work accurately with decimal fractions; they use advanced reasoning concerning proportions, geometric representations of quantities, combinatorics and integer number relationships; and they interpret and understand formal expressions of relationships among numbers, including in a scientific context.
5	At Level 5, students are able to formulate comparison models and compare outcomes to determine best price; interpret complex information about real-world situations (including graphs, drawings and complex tables, for example two graphs using different scales); they are able to generate data for two variables and evaluate propositions about the relationship between them. Students are able to communicate reasoning and argument; recognise the significance of numbers to draw inferences; provide a written argument evaluating a proposition based on data provided. They can make an estimation using daily life knowledge; calculate relative and/or absolute change; calculate an average; calculate relative and/or absolute difference, including percentage difference, given raw difference data; and they can convert units (for example calculations involving areas in different units).
4	At Level 4, students are typically able to interpret complex instructions and situations; relate text-based numerical information to a graphic representation; identify and use quantitative information from multiple sources; deduce system rules from unfamiliar representations; formulate a simple numeric model; set up comparison models; and explain their results. They are typically able to carry out accurate and more complex or repeated calculations, such as adding 13 given times in hour/minute format; carry out time calculations using given data on distance and speed of a journey; perform simple division of large multiples in context; carry out calculations involving a sequence of steps and accurately apply a given numeric algorithm involving a number of steps. Students at this level can perform calculations involving proportional reasoning, divisibility or percentages in simple models of complex situations.
3	At Level 3, students typically use basic problem-solving processes, including devising a simple strategy to test scenarios, understand and work with given constraints, use trial and error, and use simple reasoning in familiar contexts. At this level students typically can interpret a text description of a sequential calculation process, and correctly implement the process; identify and extract data presented directly in textual explanations of unfamiliar data; interpret text and diagrams describing a simple pattern; perform calculations including working with large numbers, calculations with speed and time, conversion of units (for example from an annual rate to a daily rate). They understand place value involving mixed 2- and 3-decimal values and including working with prices; and are typically able to order a small series of (4) decimal values; calculate percentages of up to 3-digit numbers; and apply calculation rules given in natural language.
2	At Level 2, students can typically interpret simple tables to identify and extract relevant quantitative information; interpret a simple quantitative model (such as a proportional relationship) and apply it using basic arithmetic calculations. They are able to identify the links between relevant textual information and tabular data to solve word problems; interpret and apply simple models involving quantitative relationships; identify the simple calculation required to solve a straight-forward problem; carry out simple calculations involving the basic arithmetic operations, as well as ordering 2- and 3-digit whole numbers and decimal numbers with one or two decimal places, and calculate percentages.
1a	At Level 1a, students are typically able to solve basic problems in which relevant information is explicitly presented, and the situation is straightforward and limited in scope. They are able to handle situations where the required computational activity is obvious and the mathematical task is basic, such as performing one or two simple arithmetic operations with whole numbers or percentages. Students at this level can manipulate quantitative information to make it amenable to computational analysis, such as determining the total number of points earned by teams given a record of their wins and losses.

Table I.A1.8. Proficiency levels on the mathematical content subscale: *Uncertainty and data*

Level	What students can typically do
6	At Level 6, students are able to interpret, evaluate and critically reflect on a range of complex statistical or probabilistic data, information and situations to analyse problems. Students at this level bring insight and sustained reasoning across several problem elements; they understand the connections between data and the situations they represent and are able to make use of those connections to explore problem situations fully; they bring appropriate calculation techniques to bear to explore data or to solve probability problems; and they can produce and communicate conclusions, reasoning and explanations.
5	At Level 5, students are typically able to interpret and analyse a range of statistical or probabilistic data, information and situations to solve problems in complex contexts that require linking of different problem components. They can use proportional reasoning effectively to link sample data to the population they represent, can appropriately interpret data series over time and are systematic in their use and exploration of data. Students at this level can use statistical and probabilistic concepts and knowledge to reflect, draw inferences and produce and communicate results.
4	Students at Level 4 are typically able to activate and employ a range of data representations and statistical or probabilistic processes to interpret data, information and situations to solve problems. They can work effectively with constraints, such as statistical conditions that might apply in a sampling experiment, and they can interpret and actively translate between two related data representations (such as a graph and a data table). Students at this level can perform statistical and probabilistic reasoning to make contextual conclusions.
3	At Level 3, students are typically able to interpret and work with data and statistical information from a single representation that may include multiple data sources, such as a graph representing several variables, or from two simple related data representations such as a simple data table and graph. They are able to work with and interpret descriptive statistical, probabilistic concepts and conventions in contexts such as coin tossing or lotteries and make conclusions from data, such as calculating or using simple measures of centre and spread. Students at this level can perform basic statistical and probabilistic reasoning in simple contexts.
2	Students at Level 2 are typically able to identify, extract and comprehend statistical data presented in a simple and familiar form such as a simple table, a bar graph or pie chart; they can identify, understand and use basic descriptive statistical and probabilistic concepts in familiar contexts, such as tossing coins or rolling dice. At this level students can interpret data in simple representations, and apply suitable calculation procedures that connect given data to the problem context represented.
1a	At Level 1a, students can typically read and extract data from charts or two-way tables, and recognise how these data relate tothe context. Students at this level can also use basic concepts of randomness to identify misconceptions in familiar experimental contexts, such as flipping a coin.

Table I.A1.5. Proficiency levels on the mathematical content subscale: *Change and relationships*

Level	What students can typically do
6	At Level 6, students use significant insight, abstract reasoning and argumentation skills and technical knowledge and conventions to solve problems involving relationships among variables and to generalise mathematical solutions to complex real-world problems. They are able to create and use an algebraic model of a functional relationship incorporating multiple quantities. They apply deep geometrical insight to work with complex patterns. And they are typically able to use complex proportional reasoning, and complex calculations with percentage to explore quantitative relationships and change.
5	At Level 5, students solve problems by using algebraic and other formal mathematical models, including in scientific contexts. They are typically able to use complex and multi-step problem-solving skills, and to reflect on and communicate reasoning and arguments, for example in evaluating and using a formula to predict the quantitative effect of change in one variable on another. They are able to use complex proportional reasoning, for example to work with rates, and they are generally able to work competently with formulae and with expressions including inequalities.
4	Students at Level 4 are typically able to understand and work with multiple representations, including algebraic models of real-world situations. They can reason about simple functional relationships between variables, going beyond individual data points to identifying simple underlying patterns. They typically employ some flexibility in interpretation and reasoning about functional relationships (for example in exploring distance-time-speed relationships) and are able to modify a functional model or graph to fit a specified change to the situation; and they are able to communicate the resulting explanations and arguments.
3	At Level 3, students can typically solve problems that involve working with information from two related representations (text, graph, table, formulae), requiring some interpretation, and using reasoning in familiar contexts. They show some ability to communicate their arguments. Students at this level can typically make a straightforward modification to a given functional model to fit a new situation; and they use a range of calculation procedures to solve problems, including ordering data, time difference calculations, substitution of values into a formula, or linear interpolation.
2	Students at Level 2 are typically able to locate relevant information on a relationship from data provided in a table or graph and make direct comparisons, for example to match given graphs to a specified change process. They can reason about the basic meaning of simple relationships expressed in text or numeric form by linking text with a single representation of a relationship (graph, table, simple formula), and can correctly substitute numbers into simple formulae, sometimes expressed in words. At this level, student can use interpretation and reasoning skills in a straightforward context involving linked quantities.
1a	Students at Level 1a are typically able to evaluate single given statements about a relationship expressed clearly and directly in a formula, table, or graph. Their ability to reason about relationships, and change in those relationships, is limited to simple expressions and to those located in familiar situations, such as contexts involving unit rates. They may apply simple calculations needed to solve problems related to clearly expressed relationships.

Table I.A1.7. Proficiency levels on the mathematical content subscale: *Space and shape*

Level	What students can typically do
6	At Level 6, students are able to solve complex problems involving multiple representations or calculations; identify, extract, and link relevant information, for example by extracting relevant dimensions from a diagram or map and using scale to calculate an area or distance; they use spatial reasoning, significant insight and reflection, for example by interpreting text and related contextual material to formulate a useful geometric model and applying it taking into account contextual constraints; they are able to recall and apply relevant procedural knowledge from their mathematical knowledge base such as in circle geometry, trigonometry, Pythagoras's rule, or area and volume formulae to solve problems; and they are typically able to generalise results and findings, communicate solutions and provide justifications and argumentation.
5	At Level 5, students are typically able to solve problems that require appropriate assumptions to be made, or that involve reasoning from assumptions provided and taking into account explicitly stated constraints, for example in exploring and analysing the layout of a room and the furniture it contains. They solve problems using theorems or procedural knowledge such as symmetry properties, or similar triangle properties or formulas including those for calculating area, perimeter or volume of familiar shapes; they use well-developed spatial reasoning, argument and insight to infer relevant conclusions and to interpret and link different representations, for example to identify a direction or location on a map from textual information.
4	Students at Level 4 typically solve problems by using basic mathematical knowledge such as angle and side-length relationships in triangles, and doing so in a way that involves multistep, visual and spatial reasoning, and argumentation in unfamiliar contexts; they are able to link and integrate different representations, for example to analyse the structure of a three dimensional object based on two different perspectives of it; and typically they can compare objects using geometric properties.
3	At Level 3, students are able to solve problems that involve elementary visual and spatial reasoning in familiar contexts, such as calculating a distance or a direction from a map or a GPS device; they are typically able to link different representations of familiar objects or to appreciate properties of objects under some simple specified transformation; and at this level students can devise simple strategies and apply basic properties of triangles and circles, and can use appropriate supporting calculation techniques such as scale conversions needed to analyse distances on a map.
2	At Level 2, students are typically able to solve problems involving a single familiar geometric representation (for example, a diagram or other graphic) by comprehending and drawing conclusions in relation to clearly presented basic geometric properties and associated constraints. They can also evaluate and compare spatial characteristics of familiar objects in a situation where given constraints apply (such as comparing the height or circumference of two cylinders having the same surface area; or deciding whether a given shape can be dissected to produce another specified shape).
1a	Students at Level 1a can typically recognise and solve simple problems in a familiar context using pictures or drawings of familiar geometric objects and applying basic spatial skills such as recognising elementary symmetry properties, or comparing lengths or angle sizes, or using procedures such as dissection of shapes.